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Vol. 211 APRIL, 1931 No. 4

THE SCIENCE OF VOICE.*

BY

DOUGLAS STANLEY, M.S., A.C.G.I.

"The history of science is the progressive emancipation of man from witchcraft and taboo, a gradual and often painful shift from imperfect and hasty generalization to exact deduction from meticulously observed and collected data. In many fields of science the shift has been complete and revolutionary, the old ideas remaining only in surviving superstitions. . . . In the field of voice culture, however, witchcraft still remains triumphantly enthroned. Professors of the vocal art still purvey secret methods of voice placing, mysterious arts of breath control and tricks of register, much as the apothecaries of old did a bootlegging business in love philtres. . . . It remained for the students of physics and physiology, enormously interested in the recent developments of sound recording and reproduction, to make a thorough scientific inquiry into the voice and to hurl the last thunderbolt into this stronghold of superstition." [1]

To the scientific mind a state of affairs such as is depicted by this well-known music critic is well nigh incredible. But then few scientists have read any of the avalanche of books dealing with vocal technic which have been published by all sorts of people all over the world.

* Presented at a meeting held Thursday, October 30, 1930.

[1] J. W. Fay, reviewing The Science of Voice, by the author, for the Musical Observer.

The real reason for the condition of affairs depicted by Mr. Fay is not far to seek. Vocal technic has heretofore been taught almost exclusively by singers, ex-singers, coaches, accompanists, music composers and, in a few instances, throat specialists. Now, all that is really true about the subject of voice must be based not upon the several and separate sciences of physics (acoustics), anatomy, physiology, psychology, and upon the laws of music, interpretation and dramatic utterance, but upon the sum total of all of these sciences. It will be quite evident, then, that, when this question of voice is considered from only one or even two special angles, the resultant deductions will inevitably be subject to error because of a lack of inclusion of the remaining angles.

When, some ten years ago, the author decided to investigate the subject of vocal technic as a *totality*, he realized that the problem was one which demanded group work. He was most fortunate in enlisting the interest of various physicists, including S. S. A. Watkins, Drs. H. H. Sheldon and Lloyd B. Ham and also the late Dean Holmes C. Jackson, head of the Department of Physiology of New York University. The author was able to enlist the aid of various other scientists of the Bell Telephone Laboratories and of the Medical Department of New York University, and in their work, they took particular pains to coördinate their viewpoints and conduct researches jointly.

The results of our researches destroyed practically every theory held by vocal teachers all over the world, and set up an entirely different set of principles, since the old theories were based, if not upon guesswork and superstition and a strongly held but unreasoning tradition, at least upon partial science only.

In this presentation the author endeavors to give an outline of his findings. While there is still much work to be done, and a virgin field of research is still full of possibilities for further scientific work, he feels that he has already gone far enough to establish a set of fundamental principles which make it possible really to train a voice and make improvements in the student's technic which would have been completely impossible under the old era of the 'try and miss' methods.

A GOOD VOICE NOT INDIVIDUAL.

The belief that a great voice is the result of certain anatomical gifts and peculiarities of the individual possessing it has been universally held. This idea is totally fallacious. It is theoretically possible for any healthy human voice to produce sounds equal to or better than those phonated by the greatest living artist. What has, in the past, been believed to be the "natural voice" of the singer or speaker is, in actual fact, merely the *state* of his technical equipment. Vocal training should in effect consist in the actual 'creation' of an ever-improving "natural voice." The degree to which the pupil can be trained depends rather upon his tractability and innate talent than upon his natural vocal equipment.

VOICE 'STARTS' IN MENTAL CONCEPT.

The human voice is a sound-producing instrument subject to the laws of sound and governed by the laws of physiology —all scientific laws.

The voice instrument consists of (1) a *motive force*—the PRESSURE of the breath; (2) a *vibrator*—the vocal cords; and (3) a *resonator* consisting of a set of adjustable resonance cavities. When a singer (or speaker) starts to phonate, nerve impulses are sent from the brain according to the pattern or mental concept of the person, and *the mental concept* brings into action certain groups of muscles. The muscles available are those of the thorax, larynx and pharynx, and these are brought into simultaneous action. In the case of the ordinary voice, various other groups of muscles are tensed, and these interfering groups of muscles form part of the individual's *concept* of tone; this coördination of interfering muscles is also due to certain psychological inhibitions.

A fundamental requirement of the teacher's art lies in the correcting of mental concepts, and in the overcoming of inhibitions so that the interfering muscular contractions may be eliminated and the mental concept may become similar to that of the singer who is born with a 'great natural voice.' The first law for the teacher of voice will have to be that no direction may be given the pupil which pertains to the direct control of any narrow group of muscles taking part in the act of phonation. All directions to the pupil must either be

of a general nature, or must pertain to groups of muscles which are not or should not be used in the act of phonation. Thus, any direction to the pupil which pertains to the position of the larynx, the base of the tongue, the palate and so forth, or to the control of the breath is physiologically impossible of accomplishment DURING THE ACT OF PHONATION. The result of such directions is further confusion of the mental concept, which is extremely harmful technically, and a sense of helplessness which inevitably results from an attempt to accomplish the impossible.

THE RÔLE OF THE BREATH.

This leads to a discussion of the action of the breath during the act of phonation. We have said that it is the PRESSURE of the breath which brings the vocal cords into vibration.

Unfortunately, most teachers have a mental picture of a 'stream of breath' and thus an utterly absurd and meaningless concept has arisen which is widely held, and which has formed the basis of many of the current fallacies about the technic of singing. This notion, which is merely laughable to the scientist, is as follows: "Breath is voice; the stream of breath, passing over the vocal cords, becomes 'vocalized breath' and this stream of "vocalized breath" must be directed into the—here the different schools vary—nose (American school), bony masque (French school), against the hard palate (English school), up and over into the head (German school), or, merely, forward (Italian school)."

This false picture takes no cognizance of the elementary facts of physics. Sound is transmitted through the transmitting medium by means of alternate compressions and rarefactions of the air particles in all directions at the rate of about 11 hundred feet a second in air at normal temperature and pressure. In singing, there is not a stream of fast moving air—i.e. a draught; the breath escapes very slowly when the technic is good.

When the voice is properly produced, the control of intensity is a function of the laryngeal muscles, as will be explained later in connection with the question of registration. When the voice is faultily produced and the tone is 'throaty,'

the rate of expulsion of the breath is governed by the degree of constriction of the laryngeal pharynx (throat).

At the normal intensity of the voice—this is determined only when the technic is good—the breath expulsion is at the minimum. The increase of intensity from this normal intensity—M.F.—to F.F. must be accomplished by means of an increase in the amplitude of the vibrato, or in other words by means of a general increase in the periodic interpulmonic breath pressure. At M.F., the vocal cords are approximated to the greatest possible degree and the breath expulsion must be at the minimum—the increase in intensity must then be due to an increase in the breath pressure. BUT, when the singer starts to reduce the intensity from M.F. to P.P., the action which takes place should be a function of the laryngeal muscles. Thus the intensity should be decreased because of the progressive separation of the vocal cords until the whisper is reached. Then phonation ceases, and the sound is produced by means of a rush of air over the false vocal cords which actuates the adjusted resonance cavities. Thus the pure whisper cannot be phonated. The pitch is fixed for all voices. Here we have only the vowel resonation and the toneless consonants accompanied by a rush of breath.

Unfortunately there is another method by means of which the intensity of the tone may be diminished.

As will be shown later, the quality depends upon the 'set' of the resonance cavities, and this 'set' should not vary with the intensity. When the technic is 'throaty,' however, the decrease in intensity is accomplished by means of a progressive increase in the degree of constriction of the throat. Here the control of the breath becomes a function of the pharyngeal muscles, and THE BREATH EXPULSION DECREASES AS THE INTENSITY IS REDUCED.

RESEARCH ON BREATH EXPULSION.

The author directed a great deal of research upon this question of the expulsion of the breath during the act of phonation. The results of this research were very fruitful, and brought to light many vitally important facts. He also developed a method of technical criticism by means of breath-expulsion curves.

The procedure was as follows: The singer put on a French gas mask, in the eye piece of which was installed a telephone transmitter. Rubber tubing went from the mask to a spirometer and to the air. Flutter valves allowed the air to be breathed in from the outer air and out into the spirometer, but stopped the air from escaping into the outer air or out of the spirometer. The telephone transmitter, which was especially designed for such work by the Bell Telephone Laboratories, was connected to a specially designed amplifier and thence to a General Electric oscillograph. From the oscillograms (Figs. 5*a* and 5*b*) readings of the relative intensity could be obtained.

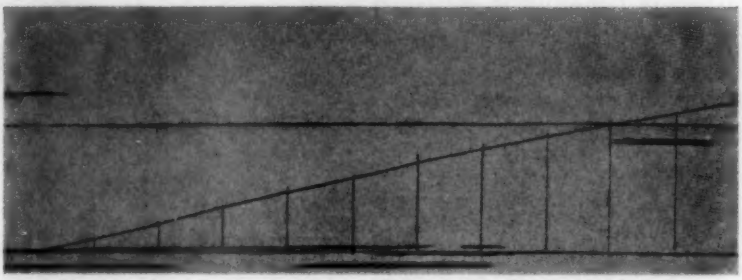
A mirror was mounted upon the shaft of the floating lid of the spirometer, and from this mirror a spot of light was reflected onto a revolving drum on which was mounted a piece of sensitized paper. As the drum revolved a second marker traced the intervals of time. In this manner readings of the volume of air expelled per second were taken. Fig. 1 shows the gas mask, spirometer, revolving drum and time marker.

The subject was instructed to sing several tones at various pitches and intensities, and on different vowels, each tone

FIG. 1.

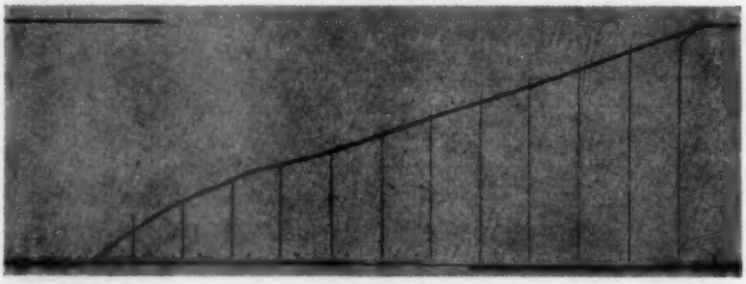
being held for about eight to twelve seconds. Figs. *2a* and *2b* show two spirometer records. Fig. *2a* shows the record of a well-produced voice with the resultant regular curve. Fig. *2b* shows a faulty voice with a more irregular curve—note the poor 'attack' with the sudden rush of breath at the inception of the tone.

FIG. *2a*.



One of the breath-output records made by the spirometer, showing a uniform expulsion of the breath.

FIG. *2b*.



Another breath-output record that is much less uniform. The vertical lines mark seconds of time.

In taking the oscillograph tracings (see Figs. 5*a* and 5*b*) the drum of the oscillograph was run relatively slowly, so that the wave form does not show. The area of such a curve gives a measure of the relative intensity, which, although not absolutely accurate, was found to be sufficiently so for our purpose. The points and hollows indicate the vibrato and not the fundamental of the tone. These tracings were also marked out in seconds. Mean results over the given period gave the average expulsion and intensity per

second. From these readings curves were plotted, one giving breath expulsion per second against intensity with pitch constant and the other breath expulsion per second as against pitch with the intensity constant at M.F. The "breath expulsion-intensity" curves were taken at three pitches, one in the lower range of the voice, one in the middle and one in the upper. These curves were also taken for different vowels.

From such curves much can be read. In fact, Dr. Sheldon was able to deduce from them a physiological (technical) criticism of a given voice without having heard it, which closely corresponded with the author's criticism which was founded upon technical and physiological observations of the actual subject.

FIG. 3*a*.

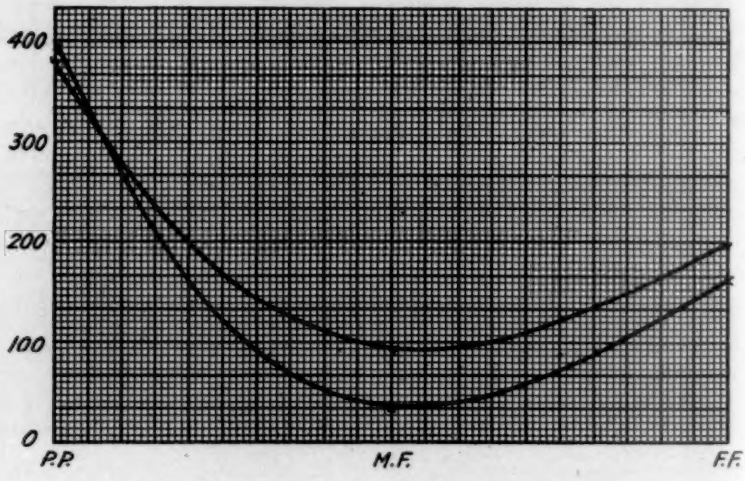


Fig. 3*a*, shows the "breath expulsion-intensity" curves for a well-produced voice, at two pitches. Note the form of the curve indicating the minimum breath expulsion at M.F., the rise in expulsion as the intensity rises and the more rapid rise as the intensity decreases to P.P.

Fig. 3*b* shows a "breath expulsion-intensity" curve for a badly-produced 'throaty' voice. Notice that the minimum breath expulsion is at P.P. and that the curve shows a regular increase in breath output as the intensity rises. Here the

control of the intensity is a function of the pharyngeal instead of the laryngeal muscles.

FIG. 3*b*.

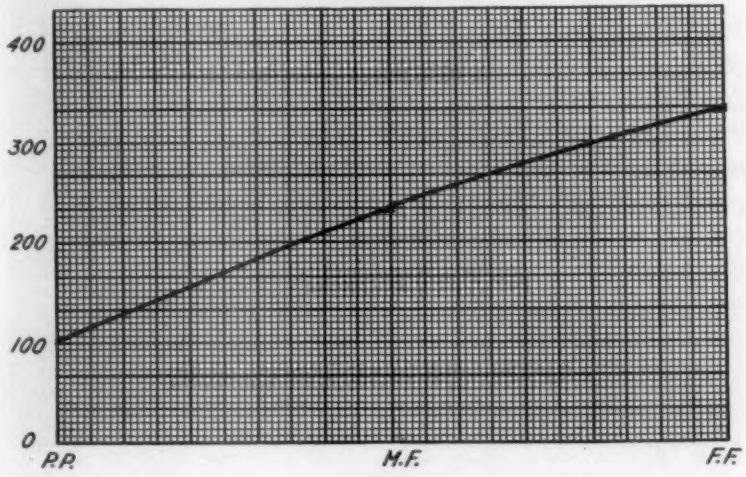


FIG. 3*c*.

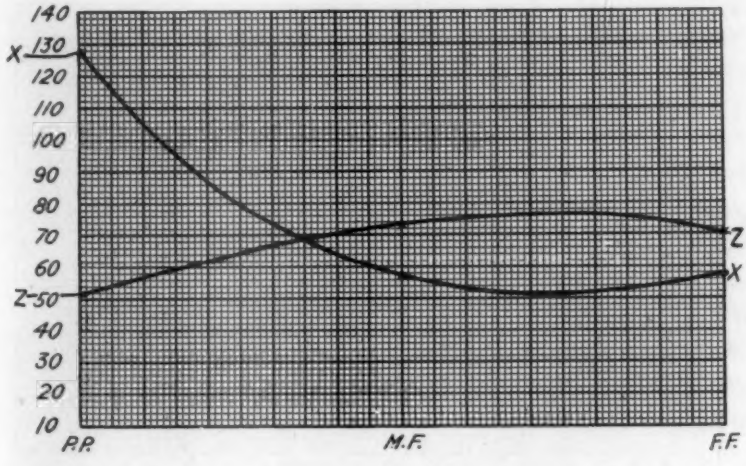


Fig. 3*c* shows two "breath expulsion-intensity" curves of a pupil in the early stages of training and after a period of some months when the technic had improved considerably—

she was still far from perfection. Notice that the curve marked *X*, which was taken in the later stage of technical

FIG. 4*a*.

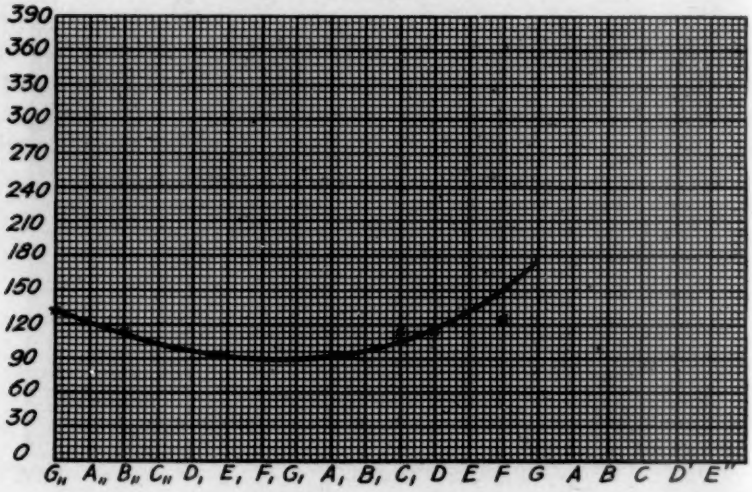
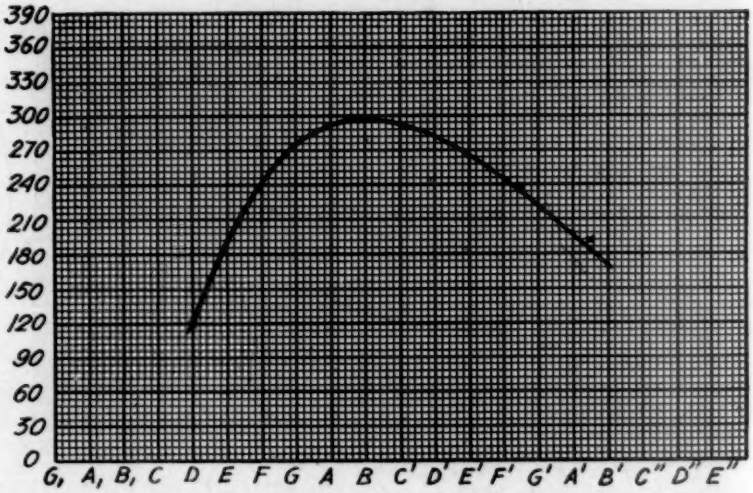


FIG. 4*b*.

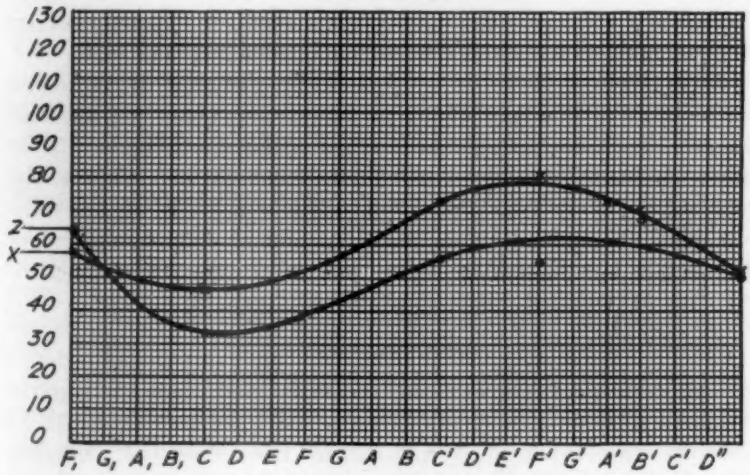


development, is beginning to show the correct form while the curve marked *Z* is inverted.

Fig. 4*a* shows the form of the "breath expulsion-pitch" curve for a well-produced voice. Notice the form of the curve showing the maximum efficiency or breath output in the middle of the range and the rise in the breath output as the pitch rises or falls. This curve is taken from the same subject who gave the results plotted in Fig. 3*a*.

Curve 4*b* was for the 'throaty' singer who produced Fig. 3*b*. Notice the absolute inversion of the curve showing the lowest efficiency at the middle of the range where the breath expulsion should be at the minimum. Fig. 4*c* shows

FIG. 4*c*.



the curves of the same lady—a soprano—who produced Fig. 3*c*. The peculiar form is due to the effect of the faulty registration which will be explained later. Notice, however, that the second curve marked *X* shows a better overall efficiency than does the first curve marked *Z*.

BREATH CONTROL FALLACY.

In all countries, and at all recorded times, there has existed an idea that the singer should learn to control his breath and that 'breath-control' is a vital part of vocal technic. Teachers who entertain this fancy even prescribe exercises for the conscious control of the rate of expulsion of

the breath, used in the act of phonation, by means of the expiratory muscles. That any such attempt on the part of the singer is abortive is evident from the statement already made to the effect that the entire act of phonation is a single act, and that direct control of any narrow group of muscles used in this act is physiologically impossible. True, the rate of expulsion of the breath can be easily regulated when the individual is not phonating, but the moment he commences to sing such control by means of the expiratory muscles becomes impossible. Furthermore the intensity is not directly controlled by the breath output when the technic is good, but it is so controlled when the production is 'throaty.' Thus, when the technic is good, an increase in breath output may either increase or decrease the intensity from M.F.

Any attempt to control consciously the rate of expulsion of the breath during the act of phonation must, in the last analysis, lead the singer to control reflexly this output by means of a progressive constriction of the throat. Thus, the teaching of 'breath-control' must inevitably lead to 'throatiness' and vocal deterioration. Psychologically, the effect of this attempt will be to make the pupil 'breath-conscious,' and thus seriously impede his sense of freedom.

THE VIBRATO.

Condemning the prevalent and universal ideas of 'breath-control' calls for the presentation of a constructive truth in place of the fallacy. What takes the place of 'breath-control'? The *vibrato.*

Observation of the great singers will show that they never produce a 'dead,' steady tone except at pianissimo. The tone fluctuates in intensity with perfect regularity, and this fluctuation of intensity, which is very wide at high intensity, is accompanied by a slight fluctuation of the pitch. The author has conducted researches into the phenomenon of the vibrato and his results may be summed up as follows: The vibrato is the result of a periodically applied impulse to all the muscles which coördinate in the act of phonation. The frequency of the vibrato is about six a second but this frequency can be increased under certain circumstances. Thus all the time the singer is phonating he is actually singing and

stopping singing very rapidly and perfectly regularly. As the intensity rises the 'on' impulse is more vigorous and the swing of the vibrato increases. Since at M.F., when the technic is good, the intensity of the tone should increase as the pitch rises nearly up to the top of the singer's range, the vibrato should increase in amplitude as the scale is ascended. Of course the vibrato decreases as the intensity drops until a point is reached at which a stream of air actuates the vocal cords, when it disappears altogether.

Now the reader must fully realize how different is this concept of an 'on' and 'off' impulse from the notion of a steady control of a stream of breath. Under proper technical conditions, the intensity is regulated by means of the vibrato. The 'on' impulse is more vigorous and the breath pressure greater, at this moment, as the intensity rises. Such an action allows of freedom of the voice producing mechanism and eliminates the risk of tetanic muscular contraction.

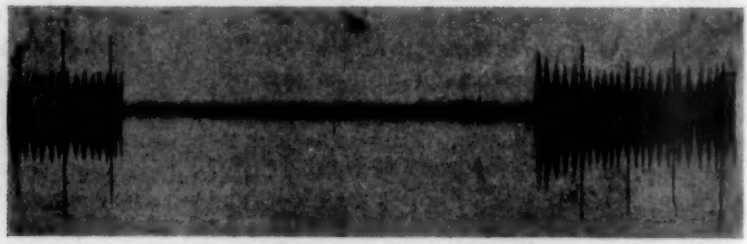
The importance of the vibrato has many ramifications. Thus all movements of the voice must be made '*on the vibrato.*' The transition of the voice from tone to tone must be made on a single vibrato. Any other method of 'moving' the voice must result in either a jerk or a slur. The singer should never under any circumstances slur. A progressively changing pitch is mere noise. Thus, in running a scale at high speed each tone is one vibrato and the singer moves on the vibrato. The speed of the singer depends upon his speed of vibrato and, when he can speed up the vibrato, he can run scales extremely rapidly. Only in this manner can the clean-cut, 'pearly' effect, which is one of the concomitants of a fine technic, be attained. When a slur down is indicated on the printed page, the singer must run down on the vibrato and not slur. The result of a correct run down is a beautiful, instead of a 'whining,' effect.

Again, the regular beat of the vibrato gives a unit of time, and allows the singer to keep perfect time and rhythm. Each note has a definite number of vibratos, and this gives an exact 'time spot.' Much more could be said on the musical side about the importance of the vibrato, but enough has been set down to indicate how vital is this phase of the technic.

The vibrato may be faulty in many ways. It may be

too slow or irregular. It may have the wrong amplitude for the given intensity. It may even degenerate into the *tremolo* —a most lamentable fault.

FIG. 5*a*.



Regular vibrato, typical of well-produced voice.

FIG. 5*b*.

Irregular vibrato, typical of poor quality voice.

FIG. 5*c*.

Tracing of a phonated tone taken at high speed and showing the form of the sound wave and the vibrato. This tracing shows one half of one vibrato.

The tremolo is produced when the resonance cavities are relatively relaxed and fluctuate during the act of phonation. Under this circumstance the epiglottis usually quivers. The tremolo often occurs when a soft very

'throaty' tone is produced and, when such a tone is swelled, the movement generally ceases. The tremolo, therefore, is practically the inverse of the vibrato. It is the result of the fluctuation of one part of the voice-producing mechanism, and is not engendered by means of a periodically applied impulse; also it tends to decrease as the intensity rises. In some extreme cases the tremolo is the result of such intense constriction that the jaw is also brought into sympathetic tetanic movement. This is a serious technical fault and is very dangerous. Fig. 5*a* shows an even vibrato from a well-produced voice and Fig. 5*b* an uneven vibrato from a badly-produced voice.

While the author has done a great deal of work on the vibrato, there is quite an open field for research in this direction. The ideal pitch and intensity variations for the perfect voice have still to be determined by the recording of relative results from many singers of all calibers. So important is the vibrato, that we believe that, when absolutely definite laws have been established, a perfect criticism of a given voice may be deducible from vibrato records alone—see Figs. 5, *a*, *b* and *c*.

When the pupil inspires in a faulty manner it is incumbent upon the teacher to show him the proper method, but such work must not be associated with the act of phonation. The training of the vibrato, on the other hand, can only be accomplished during the act of phonation, of which act it is a part. Training the vibrato is a most important phase of the teacher's art.

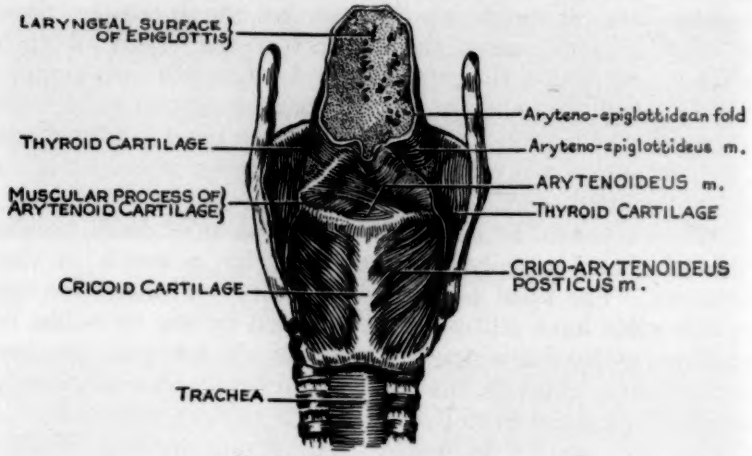
THE REGISTERS AND THE LARYNGEAL MUSCLES.

Following the discussion of the actuator of the vocal mechanism, or the motive force, the next question to be dealt with is the action of the vibrating mechanism—the larynx and vocal cords and their accompanying muscles.

Upon starting to phonate, a singer must have a mental concept of the pitch which he is about to sound. If this mental concept is faulty, or if the receiving mechanism of the ear is faulty, the pupil cannot be made to sing, although the speaking voice may be trained. If the motor response between the brain centers and the muscular system which

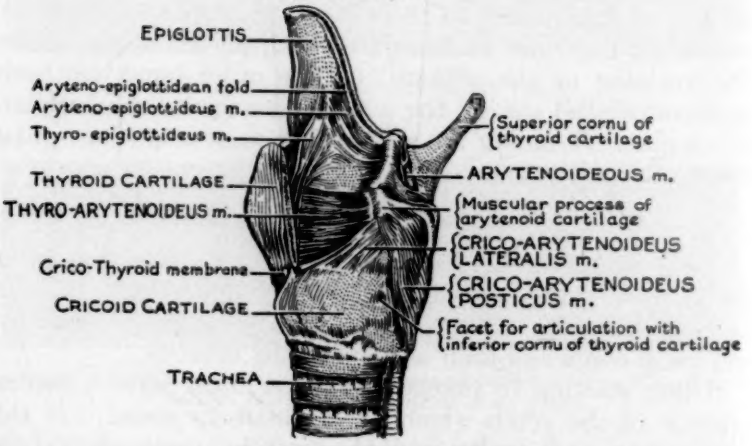
actuates the vocal organs coördinates in a faulty manner—i.e. if the technic is faulty—the singer will sing off key, but this can be rectified entirely when the technical faults have been overcome.

FIG. 6.



Muscles of larynx. Posterior view.

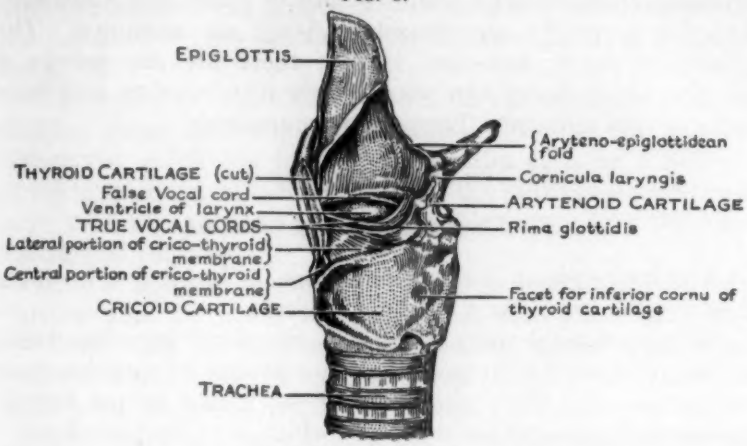
FIG. 7.



Muscles of larynx. Lateral view.

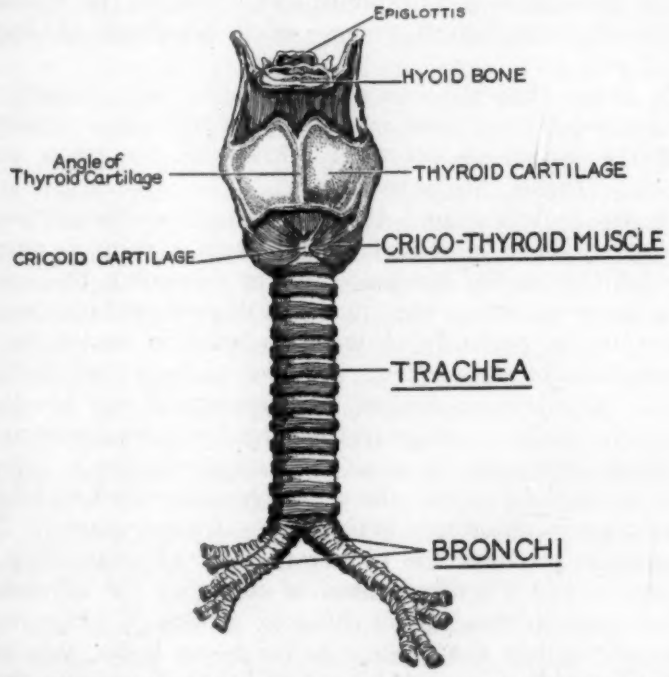
Figs. 6, 7, 8 and 9 show the larynx, vocal cords and the muscles which actuate them. A detailed description of the

FIG. 8.



Lateral view of interior of larynx.

FIG. 9.



Outline showing general view of hyoid bone—larynx—trachea—bronchi and crico-thyroid muscles.

action of these muscles would be out of place here, since such may be found in any standard book on anatomy. The important point, however, is that there are *two groups of muscles* which bring the vocal cords into tension and hold them in this tension. These two groups are:

(*A*) The arytenoid muscles

(*B*) The crico-thyroid muscles.

The latter group (two muscles) are far larger and stronger than are the arytenoids (seven muscles).

When a mental concept initiates the nerve impulses which are supplied to the larynx, these two groups of muscles come into action, and the vocal cords are set to the proper length, tension and position for the pitch which is to be phonated.

Let us consider what should happen when the tone is properly produced, if the singer starts from P.P., where the breath pressure is at minimum, and increases the intensity, and therefore the breath pressure or the amplitude of vibrato, to F.F.:

In order that the cords may retain their tension, the muscles which hold them must take on additional tension to resist the increasing pressure. Otherwise the pitch would change. How is this to be done?

Under ideal conditions, the lighter muscles—the arytenoids—should take on increased tension until a point is reached at which they would overload, and, at this point, the tension of the larger muscles—the crico-thyroids—must be increased.

Let us be perfectly clear as to what is meant by the phenomenon of *registration*. As long as the vocal cords are held in their tension against the pressure of the breath by means of additional tension on the lighter muscles-arytenoids—the singer or speaker is using the "*upper register*." As soon as the stronger muscles—the crico-thyroids—start to take on added tension, the singer is using the "*lower register*." Thus registration provides the proper method of controlling the INTENSITY, and it is not a means of changing the adjustment of the vocal mechanism for different groups of tones—i.e. it does not regulate the pitch. As far as we know, this latter theory has always been held and no other alternative theory

has been advanced. Hence pupils are either told to use only one register, or they are made to change register at a certain low pitch.

The point at which the tension on the stronger group of muscles must be increased changes with the pitch. The higher the pitch the louder will be the tone at which the added tension on the crico-thyroids—the lower register—must come in. Fig. 10 gives a schematic diagram of how the registers should function.

At the bottom of the range the upper register is a mere whisper and the lower register comes into play for a very soft tone. At the top of the range the upper register coordination will hold the tension of the cords up to practically the loudest tone that can be produced.

Now there is a very interesting point in connection with the vocal mechanism of the two sexes. The vocal organs of the male and female are exactly similar in structure, except that the male larynx is somewhat larger than the female. Both sexes have two, and only two, definite and distinct registers. Roughly speaking, when voice teachers speak of the 'chest voice' of the female they mean the lower register, and when they speak of the 'head voice' they mean the upper register. Again, when they speak of the 'falsetto' in the male voice they mean the upper register, and when they speak of the 'chest voice' they mean the lower register.

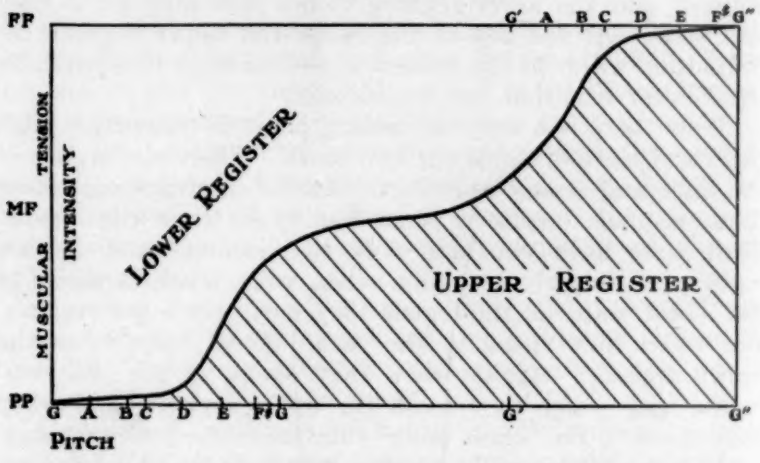
Unhappily the usual procedure is to train the upper register of the woman and neglect the lower entirely, and, inversely, to train the lower register of the man and shun the falsetto. Nevertheless both registers are equally important in both sexes, and the neglect of either one is absolutely destructive. When, as sometimes occurs in the case of a fine natural technic, the registers are naturally perfectly coördinated, the problem does not arise. Such coördination, however, only occurs in a few instances in a generation, and, in all other cases, the register question is probably the most vital phase of vocal technic.

FAULTS OF REGISTRATION.

When the voice is imperfect, various forms of faulty registration may occur. Either one or both of the sets of muscles which actuate the registers may be weak and un-

developed. In this case one of two things will happen when the singer starts to swell the tone. Either there will be a definite break at a certain point of intensity, or the lower register will come in before the arytenoid muscles have taken on their proper degree of tension. When this phenomenon occurs, we have the case of what the author designates '*mixed registration.*' This condition is very injurious and dangerous, and no vocal development in any individual is possible until the registers have been totally separated.

FIG. 10.



Reference to Fig. 10 will indicate that the maximum intensity of the pure upper register is very soft for the low pitches, and it is here that most of the vocal troubles start. These lower and even lower middle tones are too soft to be usable when the pure upper register is functioning. The inevitable result will be that the singer (this applies rather to women's voices) who does not use the lower register will force her voice even when she is singing extremely lightly. She will actually always use mixed upper registration when her lower register is undeveloped in the lower and middle part of the voice. This fault is very prevalent in the speaking voice. When the upper register is used for speech the pitch should be extremely high if the voice is not to be forced and strained and, since such a high pitch is impractical, mixed registration inevitably results.

Mixed registration, which is always associated with extreme 'throatiness,' engenders serious confusion of the muscular action in the larynx. This condition is, therefore, very harmful, and it is particularly pernicious in view of the fact that the singer does not feel the strain because the tone produced is not forceful or loud.

What an unfortunate thing it is that practically every teacher still works from the middle of the voice and endeavors to 'build on the soft' and to extend the range up and down from the middle! These middle tones are the hardest part of the range to sing properly since exact muscular balance is required if they are to be produced technically correctly and pressure on the upper register always results in mixed registration and 'throatiness.' The effect of such work is often not only to destroy faulty voices but even to break up the coördination of the registers of the world's greatest singers. This is the chief reason why most of the great artists lose their voices when they should be in their prime.

ISOLATION OF REGISTERS ESSENTIAL.

In training the voice properly the first step should be the *isolation of the two registers*. The pure upper register can generally be found at first for the very high tones, where the singer can phonate vigorously in the pure upper register adjustment. The pure lower register can be found first for the very low tones. The purification of one register depends upon the isolation of the other. Work should generally in the early stages be somewhat concentrated upon the weaker register which is, in most cases, the lower register for women and the falsetto for men.

The lower register of a woman sounds crude and ugly at the start. The uncoördinated lower register is used exclusively by 'coon-shouters.' It is interesting to note that, although the sounds produced by these 'performers' are of a most objectionable quality, they do not lose their voices as do the sweet, "light-voice" singers. As will be shown later, the quality depends, in the last analysis, upon the adjustment of the resonance cavities and not upon the registration. This crude lower register must be carried all the way up and developed, in spite of its harsh quality, in the early stages,

and the effort which it is necessary for the singer to exert in order to accomplish this end. The resonance cavities must be held 'patent' during this process and the singer must never be allowed to produce a 'throaty,' mixed, lower register tone.

The pure upper register must be carried down and the greatest pains must be taken not to allow the pupil to thicken or mix the quality. All singing must be performed, in the early stages, as vigorously as possible for the reasons which will appear later, but, when the pure upper register is retained for the lower tones, the actual maximum intensity will be extremely soft for these tones.

The coördination of the registers, when the muscles which govern them are fully developed, will be dealt with after the question of resonance adjustment has been taken up.

Except in the case of a phenomenal natural voice, the male falsetto is always either very weak or mixed. The weak falsetto sounds silly and effeminate and the mixed falsetto (which is the basis of the technical equipment of the 'throaty,' light tenor) extremely 'throaty.' Where the falsetto is weak the process of developing it is not such a difficult matter. Care must, however, be taken, during this process of development, in order that we may be certain that apparent development is not the result of mixture. A mixed falsetto is often louder than is the pure register, and this is especially true of the lower tones. A pure falsetto takes a great deal of breath, since it is the P.P. tone when properly resonated and this also is especially true of the lower tones. It must be carried up and down over practically the entire range of the voice, and it becomes quite full and loud for the highest tones which can be reached. As the falsetto develops and improves technically, the false, effeminate quality disappears and its quality becomes indistinguishable from that of the lower register when the resonance adjustment is perfect.

The lower register, even of a man, can never be pure until the falsetto, or upper register, has been isolated and developed.

The registers must not be coördinated until full development has been attained and then great caution must be exerted in order to avoid mixed registration.

Men lose their voices, in most cases, because of mixture of registration and the resultant 'throatiness,' and also because the phonation of the high tones without the upper register coördination subjects the vocal mechanism to severe strain. *This strain is worst when the upper tones are sung softly.*

THE RANGE OF EVERY PROPERLY PRODUCED VOICE SHOULD BE AT LEAST THREE OCTAVES, AND SOMETIMES NEARLY FOUR MAY BE REACHED. Incorrect registration cuts off at least one octave, and sometimes considerably more, from the range.

The author is most anxious not to be misunderstood when he discusses the falsetto. He really means the sound which, when untrained, resembles a woman's voice. The use of the falsetto in the proper sense of the term leads to the fullest possible tone, and not to the soft, meaningless high tones of the light or counter-tenor. When the singer produces this light, 'falsetto-sounding' tone which we, unfortunately, hear so often over the radio and in the talkies, he is not using falsetto but mixed-falsetto. The proper training of such a subject is no simple matter, since this mixed registration must be broken up and the pure falsetto established before any development should be attempted.

With regard to power; any properly produced voice can fill the largest hall or opera house. The phonation of full high tones is largely dependent upon registration. A pure, lower register tone cannot be forced, and the singer whose voice has been properly trained will never strain his vocal organs. Forcing the voice is a concomitant of mixed registration which results from any endeavor to train the voice up and down from soft, middle tones.

As the author has stated, all speech must be in the lower register unless quite a special effect is called for. The training of the speaking voice, then, depends upon the isolation of both registers, and the range of the speaking voice rests with the development and coördination of *both* registers.

THE RESONANCE CAVITIES.

The author has stated that the vocal cords stretch to a certain length, tension and position, whereby the pitch is determined. They also segment and give rise to a series of harmonic overtones. There are present, therefore, the fun-

damental, the first overtone having a frequency of twice the fundamental, the second overtone having a frequency of three times the fundamental, etc., up to perhaps twenty or thirty components.

The vowel sound, or timbre, is determined by the picking out or *resonating* of certain bands of frequencies. These bands do not depend upon the fundamental. Much work has been done on the vowel frequencies, and average values have been determined for the normal voice, with considerable accuracy. It may be as well to mention here that, as far as we know, no attempt has been made to establish the ideal vowel frequencies for the perfectly produced voice. The author hopes some day to be able to conduct a research on this subject.

Any given tone has a certain amount of energy, which is distributed among the fundamental and the various overtones. According to work which we have done and also according to a paper published in the Zeitschrift fur Physik, Vol. 47, No. 3 and 4, 1928 by W. S. Kasansky, and S. N. Reschevkin, entitled "*Investigation of the Timbre of the Human Voice, and of Musical Stringed Instruments*," the energy in the tone is concentrated in the fundamental and the two or three vowel-frequency bands when the quality—i.e., the technic—is good. When the technic is faulty, the energy in the tone is distributed over a large number of frequencies.

It is, therefore, evident that we must have a means of *picking out* the fundamental and the vowel frequencies, from the series of frequencies initiated in the larynx. The method by which this is accomplished is through the accurate adjustment of the resonance system of the voice.

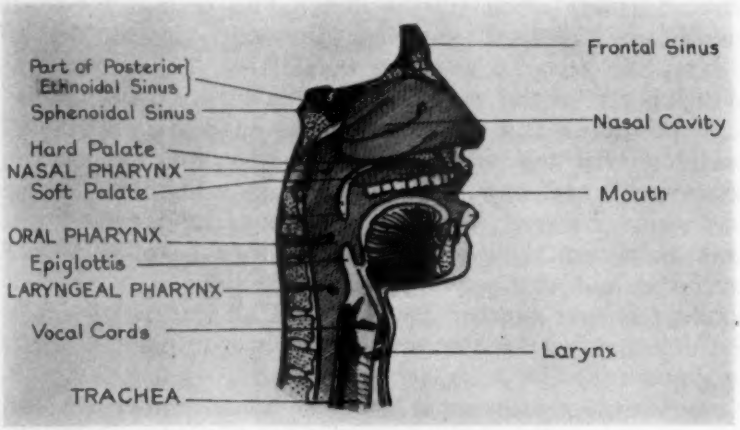
Let us consider all the possible cavities which could act as resonators. It is most important to realize that, since the accurate resonating of the tone must depend upon an extremely delicate adjustment of the resonance cavities, non-adjustable cavities must, of necessity, be of very minor importance. The possible resonance cavities are:

1. The *Trachea* and *Bronchi*—the cavities below the larynx. These are non-adjustable cavities. See Fig. 9.

2. The *Laryngeal Pharynx*—the throat. A widely adjustable cavity situated immediately just above the larynx. See Fig. 11.

3. The *Oral Pharynx.* An adjustable cavity situated behind the mouth and above the laryngeal pharynx. See Fig. 11.

FIG. 11.



4. The *Nasal Pharynx.* An adjustable cavity situated behind the nose and immediately above the oral pharynx. See Fig. 11.

5. The *Nasal Cavity.* The cavity of the nose which is highly damped with loose folds of mucous membrane and is also non-adjustable. See Fig. 11.

6. The *Sinuses.* Small, non-adjustable cavities. See Fig. 11.

7. The *Mouth.* A widely adjustable cavity. See Fig. 11.

While the non-adjustable cavities must have some effect, provided that their natural frequencies are in the neighborhood of the fundamental frequency or of those of any of the overtones, this effect is not controllable and does not enter into our technical problem. Their action is probably to give certain, very slight, personal modifications of the general tone quality. The exception to this statement is found in the function of trachea and bronchi, since here we can have a resonance system with the vibrator situated *in* a complex cavity one part of which—the laryngeal pharynx or the pharynx in toto—is adjustable. Realizing that the entire sound producing system must act as one unit, these cavities

can, for this reason, have a very definite effect. That this effect is of great importance for the production of the lower tones, will be shown later. The mouth can be, and generally is, shaped for the resonating of the vowel. Let us consider what happens when this is done: The boundaries of the mouth are the teeth and lips, the hard palate, the soft palate, the tongue, and the epiglottis. Thus, when the mouth is set for the vowel, all these members must assume a fixed position. But—the tongue, the epiglottis and the soft palate govern the shaping of the pharyngeal cavities—the pharyngeal, oral and nasal pharynx (see Fig. 11). Let us emphasize, however, that the vowel and fundamental must both be accurately resonated, if the tone is to be properly produced and the quality good, and, since they are independent of one another, the fixing of all the cavities, when the mouth is set for the vowel, renders it impossible for the fundamental to be accurately resonated.

By actual experiment it can be demonstrated that a singer who uses his voice correctly can move his jaw and lips into many varying positions and retain a constant vowel and fundamental. Also such a singer can hold his mouth and lips in constant position and produce practically all the vowel sounds.

MOUTH SHOULD NOT BE RESONATOR.

The deduction is, then, obvious. WHEN THE TECHNIC IS GOOD THE MOUTH IS NOT USED AS AN IMPORTANT RESONATOR OF THE VOICE. THIS FUNCTION BELONGS, PROPERLY, TO THE PHARYNGEAL CAVITIES. One of the principal differences between good and poor vocal technic in speech and in singing lies in the degree to which the mouth is used. One of the chief aims of the vocal teacher must be to throw the mouth out of action and to bring the pharyngeal cavities into play. Inter-adjustment of the pharyngeal cavities can provide a complex resonance cavity which is able to provide accurate resonance adjustment for both fundamental and vowel over the entire range of the voice.

It is, of course, evident that a resonance cavity must be SET if it is to act as an efficient resonator, since a relaxed fluctuating cavity would give rise to a tone of fluctuating quality. If the jaw is set—and it must be set if the mouth

is used as the vowel resonator—the tension so engendered will be communicated directly to the larynx, thereby causing serious confusion in the laryngeal action, i.e., interference with the action of the registers. Here, then, is another reason why the mouth should not be set for the vowel—the jaw must be relatively relaxed, if the laryngeal muscles are to act efficiently.

MUSCLE TONE.

Every set of muscles in the body has opposed to it a group of antagonistic muscles. No muscle in the human body can be entirely relaxed when the person is alive. The degree of reflex tension in any muscle, which has not been called into conscious action, is designated as the muscle tone. Every muscle in the body has a certain degree of reflex tone varying with different individuals. When one sits in any given position, however relaxed he may be, he is maintained in this position, by reason of this reflex tone. It is only when one starts to move that a conscious nerve impulse brings the muscles which actuate the members to be moved into tension.

At a certain degree of tone, the slightest increase in the nerve impulse will bring the muscle into tension. An ideal state of tone is one in which all the "slack" is taken out of the muscle so that the slightest possible nerve impulse will bring it into tension. The ideal condition for the singer or, indeed, for the athlete, or anyone else performing any physical action in which skill and delicate adjustment is necessary, is one in which the entire muscular system of the body is in this state of exactly the ideal degree of tone. Only the muscles which should be used in the performance of the act, should come into actual tension.

Unfortunately, the average individual maintains either too high or too low a degree of muscle tone. In the former case, the reflex nerve impulses cause many muscles which are not being employed in the performance of any desired action to come into tension. Under such circumstances, when this subject desires to move, he is forced first to relax the improperly tensed muscles, and then to bring into action the desired groups of muscles. This leads to great inefficiency of action, and produces considerable fatigue. It also tends

to impede any delicate muscular adjustments which he may desire to make.

When the state of tone is too low, it becomes necessary for the subject, when he desires to move, to send a conscious nerve impulse which will first take out the "slack," and then bring the muscle into tension. This leads to an inefficient, slow response, and is also inimicable to accurate adjustment. It is obvious that one of the teacher's greatest problems is to develop in the pupil the ability to maintain the correct degree of muscle tone. The over-tensed subject must be relatively relaxed; the over-relaxed must be relatively tensed.

It is most unfortunate that, in the teaching of the technic of both singing and playing the piano, and, as far as we know of all the other technics, there has been so much talk about 'relaxation.' Absolute relaxation is impossible. *Too high a degree of relaxation is undesirable*, and the muscles which should be used must come into absolute tension. The muscles which should not be used, and which interfere with the act, should indeed be relatively relaxed. The problem, however, lies rather in the strengthening of the muscles which should be used, and, in practice, the over-relaxed pupil is apt to present the harder problem. This is especially true in view of the fact that the placid, unmusical, undramatic individual is apt to be over-relaxed. The artistic, talented pupil is apt to be rather tense and neurotic. Generally speaking it is easier to reduce than to increase the general state of muscle tone.

TENSION ESSENTIAL ON EXTENSOR MUSCLES OF THROAT.

When the throat is in a normal condition of rest (i.e., when the individual is not singing), it is relatively closed. There are two sets of muscles which govern the laryngeal pharynx—the throat—the constrictor muscles which hold it closed, and the extensor muscles which hold it open. Thus, when the individual is not singing, the constrictor muscles are in a state of relatively high tone. Generally, then, the tendency will be for these muscles to come into tension when the conscious nerve impulse generated for the act of phonation, is applied.

Our problem is to provide the mental concept which sets the pharyngeal cavities in such a position or shape that they will pick out, or resonate, the fundamental and the vowel. The implication is obvious. The extensor muscles of the pharynx must be brought into play, and they must be tensed to such a degree that they hold the entire cavity patent in the required position, at any intensity at which the singer may desire to phonate, and, at the same time, overcome the antagonistic action of the constrictor muscles.

A 'throaty' tone is produced when the mouth is used as the resonator, and the throat, which has collapsed, is held constricted when the act of phonation is initiated. Nearly every vocal fault is associated with some such action. With the exception of the few great voices of the world, every singer brings these constrictor muscles into action, in a more or less pronounced degree.

The constrictor action of the pharyngeal muscles is most pronounced, in the early stages, when the pupil is singing a soft tone very gently. In order that this antagonistic constrictor action may be overcome, it is essential that the beginner should be made to sing *very vigorously*.

In view of the fact that *the act of phonation is a single coördinated act*, bringing into play diverse groups of muscles—i.e., the muscles of the thorax, larynx and pharynx—it is *impossible for the singer to perform any ONE part of the act gently*, and obtain the proper tension in the other muscles which are used in this act. Since, in the early stages of training, the lower part of the upper register is very weak, loud singing is only possible in this register for the high tones. When singing the lower tones in the upper register, the breath expulsion will be very high, provided that the laryngeal pharynx is not constricted. The lower register may be sung as vigorously as possible over its entire range.

In summation, the teacher's problem is to overcome the action of the antagonistic constrictor muscles of the laryngeal pharynx, and to bring into play the extensor muscles. This can only be done when the pupil is instructed to sing very vigorously, since he will, in the early stages, inevitably, tend to actuate reflexly the antagonistic constrictor muscles when he sings gently. *Vigorous singing is only possible, in the early stages, when the pure lower register has been isolated.*

Since the extensor muscles of the throat must be in action for the soft just as much as for the loud tone, and since the breath expulsion is far higher for a soft than for a loud tone, and thirdly, since the vocal cords must be held in tension against this rapid stream of air, which must be generated by a high breath pressure, it is harder and requires more actual work for the singer to phonate a properly produced soft than a loud tone. This is another reason why the beginner must not be allowed to practice soft singing.

RESONANCE ADJUSTMENT FOR FUNDAMENTAL.

The quality of the tone depends upon the energy distribution in the tone, and upon the vibrato. The energy distribution in the tone, in turn, depends upon the "set" of the resonance cavities. Our problem is, then, to obtain this proper "set." Despite the fact that the exact position of the resonance cavities depends, in the last analysis, upon the mental concept of the tone which the singer desires to produce and is actually a *reflex* action, provided that the pharyngeal cavities are used as the adjustable resonators of the voice, there are certain general rules for the correct resonating of different groups of tones throughout the range of the voice—i.e., for the resonating of the fundamental. The resonance cavities of the voice do not, of course, act according to the laws which govern cylindrical tubes, but, as an approximate generalization, it may be stated that as the volume of air inclosed in the cavity must become greater as the pitch drops, and vice versa—i.e., the higher the tone, the smaller must be the cavity employed for the resonating of the fundamental.

We stated that the laryngeal pharynx should never be constricted. This cavity will, therefore, always be a vitally important part of the resonance system. Thus the smallest possible cavity which could be used must be the throat, or laryngeal pharynx, per se. This arrangement, then, must provide the resonator for the highest tones in the voice. Slight changes in the size and shape of the laryngeal pharynx per se. will allow of exact adjustment for the high tones over quite a wide range. This fact is interesting because so many persons have held that the upper tones should be 'head-tones.' This would imply the use of a relatively large

cavity for the upper tones, which is in diametrical contradiction to the laws of physics. In practice, any attempt to 'feel the upper tones in the head,' merely results in constriction of the throat and in the ultimate elimination of the upper tones of the voice. This fallacy is another contributing cause of the lamentable lack of range which we find with most singers. Incorrect resonance adjustment very often cuts off from five tones to an octave from the top of any voice.

For the middle tones a longer cavity is required, and here the laryngeal pharynx plus the oral pharynx, and then—descending the scale—the laryngeal pharynx plus the oral pharynx, plus the nasal pharynx, must be used. When a point is reached at which the maximum size of the pharyngeal cavity in toto is insufficient to resonate the tone, the cavities below the larynx—the trachea and bronchi—must be added.

As we have stated, when the resonance adjustment is correct and both registers are developed, the range of the human voice is from three to four octaves. Faulty registration and resonance adjustment can curtail the range fifty per cent. or even more, and also reduce the power to an incredible degree.

COÖRDINATION OF THE REGISTERS.

In discussing the registers the author said that he would show how to coördinate them when he had discussed the question of resonance adjustment. This problem is quite a simple one. When any given tone is sung in the upper register, with perfect resonance adjustment, and merely swelled, at a certain point of intensity, the lower register will come into play, *provided that the resonance cavities are held patent.*

The process of training a voice, then, becomes one of purifying the registers, and obtaining the proper resonance adjustment for each register separately. When this result has been obtained, the coördination of the registers presents no further problem—it becomes a reflex action. The quality of the tone does not change when the register changes, provided that the resonance cavities are held patent, and, finally, the singer whose voice is technically advanced is himself unable to tell the exact point at which the lower register is added.

WIDELY HELD FALLACIES.

Before leaving this question of the proper 'setting' of the pharyngeal resonance cavities for the fundamental and vowel, and the vital importance of the tension on the extensor muscles of the laryngeal pharynx—the throat—it might be as well to cite what has been said on this subject by most of the vocal teachers all over the world. The general direction they have given to their pupils is as follows: "Relax your throat, and get your voice as far away from the throat as possible into the head (or nose or against the hard palate)." Criticism of such a statement seems redundant, but in view of the fact that this is typical of the directions given to thousands of pupils, it is felt that it must be discussed here.

Firstly, the voice is initiated in the larynx, and the only manner in which the vocal student could "get his voice out of the throat" would be by amputation of the larynx! Secondly, we have shown that relaxation of the throat is physiologically impossible, and that any attempt to accomplish this end would result in the constriction of the laryngeal pharynx when the vocalist started to phonate. Often the unfortunate is then adjured to "Open the throat!" No one possesses such localized control over any *part* used in the act of phonation, and to direct anyone to "open the throat" and at the same time relax it, is a direct contradiction in terms.

THE ATTACK OF THE TONE.

A most important phase of technic lies in the proper method of starting or 'attacking' the tone. When the attack is correct, all the groups of muscles involved in the act of phonation must come into tension simultaneously: i.e., the muscles of the larynx must stretch the cords, the muscles of the pharynx must set the resonance cavities and, *at one and the same instant*, the breath pressure must be applied. If any one of these components of the act of phonation lags, the entire coördinated act will inevitably be maladjusted in each and all of its phases.

If the singer takes a careful breath and holds it, before he starts to phonate, the muscles of the thorax are set, and cannot, therefore, be used efficiently for the creation of the necessary breath pressure. Actually, the breath should be

inspired and *immediately* used to create the breath pressure necessary for starting the tone.

If the singer makes any attempt to set the pharynx before the moment of attacking the tone, constriction will inevitably result. On the other hand, if the throat is relaxed at the moment of attacking the tone, the extensor muscles cannot come into action, and the throat can never be brought into its proper posture after the initiation of the act of phonation. In this connection it may be as well to mention the '*coup de glotte*' which was, at one time, so strongly advocated by teachers of singing. The click resulting from this form of attack is due to the forcing open (to some slight degree) of the laryngeal pharynx, and not, as the advocates of this form of attack believed, to the bringing together of the vocal cords. Hence the '*coup de glotte*' leads only to "throatiness."

Before commencing to sing, the pupil must be directed to hold his body in an optimum condition of tone, neither tensed nor relaxed, to hold clearly in his mind an absolutely clean-cut mental concept of the pitch and vowel (timbre, or quality) he is about to sing, and then to initiate the entire act of phonation as one single coördinated act.

PHRASING, RHYTHM AND ACCENT.

The fact that the mental concept of pitch and quality must be clearly defined if the tone is to be properly attacked, leads us to another parallel phase of the subject: The artist sings in musical phrases and not by the single tone. No one can be an artist unless he is able to conceive an entire musical phrase as one single unit. Thus, the artist has a clean-cut mental concept of the entire phrase he is about to sing before he attacks the first tone. This ability to conceive groups of tones is a vital part of the innate talent for singing in the individual. If it is lacking, the pupil may be able to learn to sing single tones acceptably, but he can never become an artist. This capacity must be nurtured, and developed by the teacher.

Any musical phrase is as much dependent upon rhythm and accent, as upon variations of pitch. Thus, the artist must also have a perfect sense of rhythm, and be meticulously exact with regard to the details of time. It may be as well

to mention here that the accent is made by means of an especially vigorous vibrato. Both accent and rhythm are, in the last analysis, largely dependent upon vibrato. Only too often singers are slovenly with regard to the musical side of their art. The teacher cannot be too particular in this phase of his work.

NATURAL PITCH, QUALITY AND POWER.

In the early stages of training the pupil's natural voice is often so obscured by technical faults that it is impossible to tell even what type of voice he actually possesses. The teacher must under no circumstances attempt to guess what the natural timbre, range or power of the beginner's voice should actually be. As the development and coördination of the muscles improve, the range and power of the pupil's voice develop, often to an absolutely incredible degree, and the natural quality commences to reveal itself. At a certain stage in the development, the full range of the voice has been established (this is at least three, and sometimes nearly four octaves), and, at this stage, the natural middle of the voice and, therefore, its calibre should have been discovered.

The natural power of the voice is not fully revealed until the technic has reached a very high state of perfection. A voice may, indeed, continue to grow for many years. There is, however, a certain normal intensity, or mezzo-forte, when the voice has arrived at a certain degree of technical development (i.e., when the "breath-expulsion—intensity" curve shows the correct form), at which the voice is produced at maximum efficiency. From this normal intensity gradations must be made. The small voice will sing far more softly at pianissimo than will the normally powerful one; the fortissimo of the powerful voice is, obviously, far greater than is that of the small. While the powerful voice should possess a much wider range of intensity variation, the singer whose voice is powerful should, under no circumstances, attempt to emulate the pianissimo of his lighter voiced confrere. One of the most fatal mistakes of many uninformed vocal teachers lies in their endeavors to build all voices from an 'absolute,' and more or less equal, pianissimo, or whisper. Any such method will inevitably develop extreme 'throati-

ness' in every pupil and will, in time, entirely destroy even the finest natural voice.

When the resonance adjustment has reached a fairly high degree of accuracy, the natural quality of the voice is revealed. This is peculiar in each individual, and depends upon the size, structure, and texture of his vocal organs. From this natural quality (which must be found, since it is never revealed until the technic is comparatively far advanced), all color changes must be made.

COLOR AND EMOTION.

All the emotions which are humanly conceivable can be expressed through the medium of the human voice. The means we have of expressing emotion lie in what is designated as *color*.

The vowel sound is determined by the resonating of two or three bands of frequencies. By means of slight changes in the positions of these bands, and of the preponderance of one over the other, modifications can be made in all the vowels. Such modifications can be made over a very wide range, with the vowel still remaining intelligible. For example; when one enunciates the word 'sleep,' with a calm, soothing import, the quality of the 'E' vowel is 'dark.' On the other hand, when one enunciates such a word as 'demon' or 'fiend' as an expression of anger, he is still using the 'E' vowel but, in this case, the quality or color is "bright." The two "E's" in the examples cited are of vastly different quality, but both may be designated as the vowel 'E.' Probably, the bright 'E' is determined by the preponderance of the higher vowel frequency band, and the dark 'E' by the preponderance of the lower. Any vowel may be thus modified over a wide area, and herein lies the basis of our ability to express with, or *color*, the voice.

Minute changes in the size and shape of the pharyngeal resonance cavities will give rise to the infinite variety of vowel modifications at the disposal of the great artist. When the mouth is used as the vowel resonator, delicate inter-adjustments become impossible, and the singer, or speaker, with such a technic is utterly unable to color his voice in any true sense of the word.

While the single syllable may be thus colored, it is, under normal circumstances, the musical phrase, or the sentence, which should receive its own peculiar color—this color, modifying each and every vowel sound in the phrase or sentence.

THE CONSONANTS.

Every phonated sound must be associated with a vowel, over eighty-three of which have, we believe, been isolated in the English language alone. Vowel sounds, should be resonated by means of adjustments of the pharyngeal or back cavities. On the other hand, consonants are articulated in the mouth, and cannot be phonated, i.e., cannot be associated with a variable fundamental.

When the mouth is used as the vowel resonator, there is serious conflict between the articulation and phonation—consonants and vowels. Anyone who has visited the talking movies will have noticed the unpleasant grimacing of many of the stars of the screen. This is due to faulty technic, which forces them to cast the mouths into all sorts of positions for the vowels, and then to hastily readjust this cavity for the consonants. The enlarged image on the screen magnifies these unbecoming facial contortions. For this reason a proper technic—pharyngeal resonation of the vowel and fundamental—is of tremendous importance to the screen star. What a relief it is when the rare occasion arises on which we hear pleasant, instead of unpleasant, sounds emanating from the loud speaker, and see a face on the screen which is occupied with the expression of the emotions to be conveyed, instead of with abortive attempts at forming mouth vowels!

All consonants should be articulated with great vigor and extreme rapidity.

While discussing the subject of the talking movies, it might be as well to mention that the amplified soft tone does not sound the same as the unamplified, or less amplified, normal intensity tone. A great deal of work has been done on this subject in the Bell Telephone Laboratories, and the author has heard reproductions of soft and normal speech amplified to about the same intensity. The amplified soft sounded unnatural, and decidedly unpleasant. The ideal of the screen star will, in the end, have to be the same as that

of the great actor—he must learn to use his voice in front of the microphone in exactly the same manner as if he were speaking in a large auditorium, or on the stage. This means that his voice must be properly trained. The idea that there is a special technic for the microphone will very soon have to be cast aside. This is especially true because of rapid improvements in the electrical and mechanical design of the microphone.

One of the most unfortunate outcomes of the mechanical reproductions of sound has been the vogue for 'crooning.' Unfortunately these unnatural, 'throaty' noises, which could not have been heard in the first row of the orchestra seats before the advent of the amplifier, can be so amplified that they fill the largest auditorium. Such crooning noises represent the absolute antithesis of singing, and of all that is worthwhile in artistic vocal expression.

THE SPEAKING VOICE.

There is no fundamental difference between the singing and the speaking voice except inasmuch as (1) the vowels are far more sustained in singing than in speech; (2) there is no vibrato in speech; (3) in singing the picture is painted on a far larger scale than in speech, and, therefore, the intensity is far greater (thus the general pitch is far higher), the range is far wider and the articulation is much stronger.

In speech the 'unit' is the sentence, just as in singing the unit is the musical phrase. And just as the singing voice must move over the musical phrase in one flow of tone, broken only by the rapidly articulated consonants, so, in speech must the voice move in one continuous flow of sound over the sentence. The speaking voice should move up and down, in a definite pre-determined tune, just as, in singing, the vocalist's voice moves over the melody. In speech, however, the pitch should never be changed on a single vowel, unless it be a case of calling to someone in the distance, etc.

The unpleasant 'singing' which we sometimes hear in certain speaking voices is due to the changing of pitch on a single vowel, the over-pronunciation of the unimportant words, and the prolonged humming of the unwritten vowel which must be associated with the consonants 'M,' 'N,' and 'L.'

You will recall how the 'ham' actor carefully articulates each and every syllable and sustains the 'hum' vowel associated with 'M,' 'N,' and 'L,' which added vowel should always be phonated as rapidly as possible. He also overprolongs his vowel sounds and, sometimes, even puts a tremolo in his voice.

In enunciating a sentence the important syllables must be brought out strongly, usually on a pitch somewhat higher than that used on the less important syllables. The unimportant words—"the's," "but's," "and's," etc.—should not, under any circumstances, be brought out, and, while the flow of tone over the sentence must never be broken, they must be almost, but not quite, elided. The aim of the speaker is intelligibility. He must make clear to the audience the *meaning* of what he is saying, and express for them, by means of color (just as in singing), the gamut of all the human emotions. His aim should not be to make each and every syllable clearly and distinctly understood. Any endeavor to do this will utterly destroy the force of what he is saying, and he will not be reading into the words their true meaning.

The question of the natural pitch, power and quality, from which all modifications and changes of pitch, intensity and color are made, has been discussed with regard to the singing voice. Everything that has been said applies equally to the speaking voice, except inasmuch as the singing voice paints the picture on a far larger scale. Thus the average intensity of the singing voice is far greater than is that of the speaking voice. Therefore the natural pitch, or, in other words, the middle pitch of the speaking voice, is considerably lower than is that of the singing voice. Also, pitches which would be far too low for the singer to use effectively are of tantamount importance to the speaker.

LOW TONES MOST EFFECTIVE.

Extremely low tones in speech have an extraordinary carrying quality. This is probably due to the fact that there is actually a great deal of energy in the fundamental and also because the separation between the fundamental and the vowel frequencies is widest at the lowest pitches. Very low tones in the speaking voice, when properly produced, carry with perfect clarity to the back of even the largest hall.

Very low tones may be used in quiet passages most effectively, and, also, most striking emotional effects may be conveyed through the medium of these pitches when they are properly colored.

ESTABLISHMENT OF THE LOWER REGISTER FOR SPEECH.

The training of the speaking voice is exactly similar to that of the singing voice. The registers must be totally separated, then developed and, when totally developed and properly resonated, coördinated. The isolation of one register depends upon the purification of the other. Inasmuch as the normal pitch of the speaking voice is lower than is that of the singing voice, ALL NORMAL SPEECH SHOULD BE IN THE PURE LOWER REGISTER. The first problem of the teacher of voice is to establish the pure lower register. With a man this can only be done when the falsetto has been established and purified.

Often a given voice is found to be over an octave too high and, sometimes, it may be displaced nearly two through faulty registration. For example, the author undertook the training of the voice of a young actor, Mr. Rollo Peters, who played Romeo for about 500 performances on Broadway. His voice was in a state of considerable strain and always became very hoarse during the performance. Furthermore, it did not carry well and lacked power and vibrancy. After the isolation of the falsetto and the discovery of the pure lower register, his voice dropped well over an octave. As soon as the normal pitch had been established, the hoarseness and strain disappeared, the quality improved enormously and the power and vibrancy increased to such an extent that every word was clearly intelligible at the very back of even the largest theatre. The point of chief interest lies in the fact that the establishment of the *falsetto*, which part of the voice was worked out first, revealed the low tones. The same course of events has occurred in many cases where the author has trained the speaking voices of actors who were in vocal trouble.

In the case of a woman the establishment of the lower register tones is absolutely vital. She should speak in a pure lower register just as should a man. Actresses who are in

vocal trouble are, in practically every instance, using mixed registration, but, in the cases of non-professionals, the use of the upper register very slightly mixed may occur. Under such circumstances the voice is very weak and dangerously subject to strain if used vigorously.

In the cases of women who have been using their upper registers all their lives, the change after training is most startling. This change is similar to that which occurs when a boy's voice 'changes.' Several cases have been found where, at a certain stage in the training, the voice 'breaks' just as does a boy's, and the effect for a few weeks—i.e. until the lower register has been fully established and brought into use—is rather comical. The alteration in the middle pitch in such cases is often very striking. An apparently light soprano has, on some occasions, shown itself to be, in very fact, a deep contralto. Let it be perfectly clear, however, that, when the voice is actually a soprano, the normal centre of the lower register is quite high.

When the registration is mixed, the voice is in considerable danger. The lower register must be established as soon as possible, and the danger point is not passed until a few tones in this register are secure. The range will develop later. In this connection, the case of Miss Erin O'Brien Moore who made such a hit in 'Street Scene' in New York and London, in which city the critics made very special and most enthusiastic mention of her voice, may be cited. She was in serious vocal distress a few months before she opened in this play. Her voice became hoarse before the conclusion of nearly every performance and she was forced to visit the throat specialist at least once, and sometimes twice, a week. The isolation of the pure lower register improved the quality and power of the voice a hundredfold and ultimately all the distress was eliminated. The sprays, nasal douches, ointments and visits to the throat specialist, etc., were soon eliminated.

It is apparent that the training of the speaking voice can only be accomplished through the PROPER training of the singing voice, and, thus, through actual singing. The vigorous muscular training which is essential can only be accomplished through singing. Also it is only through the sustained tone of the singing voice that the careful readjust-

ments necessary can be made. While a good sense of pitch is a great help in speech, it is not essential If the speaking voice pupil is really tone deaf he cannot be trained, but a poor sense of pitch, which would absolutely bar him as a singer, is no serious check upon the development of a really fine speaking voice.

Misguided vocal training is fatal to the speaking voice. Only the other day the author was told by a pupil that one of the actresses in the play in which he was rehearsing was about to lose her position because of her poor speaking voice. *She was studying singing with a teacher who was training the upper register only.* Each lesson this lady takes is rendering her voice—singing and speaking—worse.

PITCH AND PROJECTING FALLACIES.

A curious notion is current that a high-pitched speaking voice denotes youth. This is absolutely untrue. The normal pitch of a given voice is the one which sounds most youthful and natural. An unnaturally high pitch produces a strained quality, and sounds worn and old. After all, a very young man may have a bass voice and a relatively young girl a contralto. Inversely, an old man may be a tenor and an old woman a colorature soprano. The age of the individual is not denoted by the pitch of his voice.

Actors, public speakers and readers are often instructed to 'project the voice to the back of the hall.' This saying is, of course, ridiculous. The voice is not a stream of something which can be projected, directed or focussed. A properly resonated tone carries to the back of the largest hall and, when intensity is desired, the entire act of phonation must be accomplished more vigorously—that is all! This meaningless notion has often had the unfortunate result of making the speaker merely shout or scream.

USE OF PITCH FOR CONVEYING EMOTION.

That some striking emotional effects may be conveyed through the use of very low pitches has already been stated. It has also been shown that the intensity should increase as the pitch rises, provided that the technic does not become 'throaty.'

The general law is that the pitch must rise as the emotion rises, and fall as it falls. When the emotions to be conveyed are very intense, the range of the speaking voice may be at least two octaves. The well produced voice, should, indeed, be utilized at least through this range. This demands a singing range of over three octaves, since the full limits, especially at the extreme top of the range, should never be used.

Certain effects may be obtained by means of high pitches phonated quietly. Such effects are very rare and should be used most sparingly. When they do occur, the actual support, or work done, must be very great and the pure upper register must be used. This last statement is true of the scream which is properly produced by means of a very high-pitched, pure, upper-register (falsetto) tone.

Certain comedy effects may be produced by both men and women by means of the use of the pure falsetto, and the ingenue may have to use such effects, but she also should normally be using the lower register or her voice will ultimately fail. This failure usually comes at just about the age at which she desires to play heavier rôles. If she has been properly trained, the registers will be coördinated, or nearly so, and there is no danger in the judicious use of the upper register.

COLOR, AND LISTENING TO ONE'S OWN VOICE.

The color of the tone, as has been stated, rests upon vowel modification. This depends, in turn, upon extremely delicate interadjustments of the pharyngeal cavities. True color effects are only possible of accomplishment when the pharyngeal cavities are used and not the mouth.

These delicate inter-adjustments are only possible of accomplishment *psychologically. They cannot be made physically through direct control of the resonating cavities.* Hence the color must be pre-conceived in singing or in speech before the singer or speaker starts to phonate. The color depends upon the emotional import of the sentence or musical phrase to be spoken or sung.

Thus the mind must conceive the 'tune' and color before the sentence or musical phrase is sounded, and this concept must be *held* until phonation has ceased. If the mind embraces any other concept during the act of phonation, the

color will be absolutely destroyed and the emotion conveyed will be subject to the voice-user's concept of the moment. Thus if he LISTENS to his voice during the act of phonation, the emotion conveyed will be either one of conceit, if he approves of the tone he is emitting, or discontent if he does not approve. Exact rectification of the pitch or quality after the tone has been heard by the singer himself—i.e. after he has started to sing it—is virtually impossible, since he hears the sound he is making and to which he is listening. Hence his concept of the pitch and quality he desires to make is lost. Therefore the singer or speaker SHOULD NEVER LISTEN TO HIS OWN VOICE.

WHAT THE "AUTHORITIES" ON VOICE WRITE.

At the beginning of this article the author quoted what Mr. Fay, the music critic, wrote with regard to conditions existing in the field of vocal teaching today. In order to indicate exactly what he meant we will quote some extracts from the writings of certain of the so-called international authorities on voice.

Little need be said of the teachers of the old Italian school of *Bel Canto*. These 'maestros' had, of course, no reliable source of information, in the realm of physics, physiology or any other science. Their writings are naturally devoid of scientific basis. Actually, these maestros merely chose individuals endowed with fine 'natural voices,' and taught them, by dint of long-drawn-out practice, to vocalize the various musical forms: Scales, runs, trills, turns, shakes, and so on.

Coming to the more modern school, Sir Charles Santley's book entitled "The Art of Singing" is, generally speaking, somewhat similar to the works from the pens of the old Italian masters. Thus Santley states: "Immediately below the vocal cords, there exists a valve; the breath should be raised to that valve, ready before the sound is required. . . ." There is of course no such valvular mechanism.

Mme. Louisa Tetrazzini, in her book "How to Sing," says: "Tone is made in the first instance . . . by the breath passing through the vocal cords. The precise quality of the tone depends, however, on the formation and disposition of the various parts of the vocal apparatus—throat, palate,

tongue, etc., through which the breath afterwards passes, before issuing from the mouth." This prima donna obviously gave information which was unscientific. She imagined that the breath became 'vocalized' after passing the vocal cords, and that this stream of sound or of 'vocalized-breath' issued from the mouth.

In her book "How to Sing" Madame Lilli Lehmann says: "The whirling currents of tone, circling around their focal point (the attack), find a cup-shaped resonating cavity, when they reach the front of the mouth and the lips, which, through their extremely potent auxiliary movements, infuse life and color into the tone and the word. Of equal importance are the unimpeded activity of the whirling currents of sound, and their complete filling of the resonating spaces in the back of the throat, the pillars of the fauces, and the head cavities in which the *vocalized breath* must be kept soaring above the larynx, and SOARING UNDISTURBED." And again—"Every vowel, every word, every tone, can be colored as by magic, in all sorts of ways, by the well-controlled play of the lips. . . ."

In this book, there is an illustration of a medial section of a human head. From a point on the soft palate, a series of red lines are drawn. The lowest of these comes out through the upper teeth. The highest, from the top of the head. At the end of each line is a note of the scale. Under the illustration is the caption "Red lines denote vocal sensations of soprano and tenor singers."

It is evident that Madame Lehmann was unacquainted with the scientific principles of her subject.

In a book, written by Thaddeus Wronski, entitled, "The Singer and His Art," indorsed by many of the leading singers at the Metropolitan, including Guiseppe de Lucca, G. Danise, Pasquali Amato (who is now teaching), Adam Didur, etc., we find such statements as: "A lack of balance in the distribution of the *resonating surfaces*, is caused by the voice becoming enclosed or confined in one of the resonating cavities." The absence of scientific knowledge is again quite striking.

In a book written by Mr. R. L. Herman entitled "An Open Door for Singers" we find: "The above mental picture will make clear, how decreasing the vibrating surface (from

the entire cord to its inner edge) would make the voice run from a chest tone, to a falsetto tone, and vice versa. . . . From a purely mechanical point of view, anyone might appreciate the difficulty of connecting, taking up and releasing, in turn, the infinitely varied sets of delicate fibers which go to the making of a tone." There is no musculature in the larynx which would allow of such an action! The thyro-arytenoid muscles lie longitudinally in the vocal cords.

Mr. Herman also says: "One gradually learns to drive a large part of the air column (now become sound) into and against those portions above, and near, the pharynx, thus releasing the vocal cords from all overwork." He too, believes in the 'vocalized-breath' notion.

Frantz Proschowsky has written a book entitled "The Way to Sing," in which is apparent a lack of knowledge of the physical sciences: "The skull represents the body of the violin, the vocal cords the string, and the breath, the bow." And again "These vibrations are directed through the form or shape assumed by the throat or vocal tube, and through the transmission of the back wall of the throat and the spine, make the bony structure of the head—skull—reverberate, or resound, or resonate." And again: "All resonant sounds have two kinds of resonance, and two only. Vowel resonance, which is not directed into the nasal cavity nor obstructed in any other way, consonant resonance which is directed into the nasal cavity, or is obstructed in other ways . . . the obvious difference between vowels and consonants in acoustic effect, is that vowels are brilliant, and consonants dull."

Mr. D. A. Clippinger has written a book entitled "The Head Voice and Other Problems." Throughout this book he condemns scientific knowledge as applied to voice! He says:

1. "Consonants must not be allowed to interrupt the continuity of the pitch produced by the vocal cords." The very character of the consonant itself implies the cessation of tone production. No consonant can be associated with a pitch; it is in fact an interruption of phonation.

2. "When the voice is free, and *resting on the breath* as it were, it seems to sing itself."

If by voice we mean tone, it is not something which can rest upon the breath, or upon anything else. The larynx is *suspended* from *the hyoid bone.*

3. "Many lyric sopranos have no chest register, and it would be a mistake to attempt to develop one. In such voices, which rarely have anything below middle C, the middle register must be strengthened, and carried down, and made to take the place of the chest voice." Elimination of the lower register will, inevitably, produce that most destructive of all vocal faults—mixed registration.

Herbert Witherspoon in a book entitled "Singing" says:

1. "The tone seems to be very near, almost IN the lower back of the nose." He seems to conceive the 'tone' as something which is put, or placed in a certain place, or position. This idea is without foundation in physical fact.

2. "As the pitch ascends, we hear less chest resonance, more face resonance, in the bones of the face, wider and higher up, than the lowest tones. Still higher in pitch, the chest resonance seems to disappear completely, the resonance of the face becoming higher and more intense, and yet more pointed. . . . The focus of the voice seems always to be IN the vowel, pronounced, in the mouth fairly well forward, but (as Porpora said), 'not too far front, not too far back.'" This statement indicates a lack of understanding of what resonance or a vowel is. He pictures the voice as something that can be focussed.

3. "Even up the resonation, paying particular attention to laryngeal position. . . ." A direction with regard to the conscious adjustment of the position of the larynx—one of the parts used in the act of phonation—is physiologically impossible of accomplishment.

He directs the reader to form the vowels in the mouth, which is, as has been shown, a harmful precept resulting in vocal deterioration. Mr. Witherspoon also states that the breath expulsion increases with the intensity. This is, as has been shown, only true of the 'throaty' singer.

All the writers quoted so far have been ex-singers, accompanists, etc., and not scientists. That they should make scientifically fallacious statements is, indeed, far from surprising.

Now, however, we come to the writings of some of the throat specialists, who have treated with the subject of vocal technic:

Thomas Fillebrown, M.D., D.M.D., in his book "Rêsonance in Singing and Speaking," advocates at great length the idea of 'breath control.' He makes the following statement: "To test the amount of breath used in prolonged vocalization, a person skilled in the art of breathing, after an ordinary inspiration, closed his lips, stopped his nostrils,and began to vocalize. He found that the mouth with distended cheeks, held sufficient breath to continue a substantial tone for twenty-three seconds." It is utterly impossible for anyone to phonate when his mouth is shut and his nostrils closed.

F. Mario Marafioti, M.D., in an article which appeared in the *Saturday Evening Post*, May 31, 1930, says: "The voice proceeds from the mouth. The lungs and all other cavities of the body, cavities of bone and tissue alike, large and tiny, and all the bones of the body, from the head to the toes, make the sounding board of the voice. Perfect relaxation of all the muscles permits vibration, like that of the body and legs of a piano. Caruso had this power of relaxation; after a full-voiced aria, he would complain of the tingling in his legs." If the vocal cords were amputated and rigidly attached to the skeleton, and if all the bones of the skeleton were rigidly cemented together, it could perhaps act as a sounding board. The animate singer's body is not a rigid skeleton and the larynx is suspended from the hyoid bone. This 'sounding-board' notion is of course acoustically impossible. A cavity is not a sounding board. Cavities not open to the vibration of the vocal cords could hardly act as resonators; especially 'cavities of bone.' Perfect relaxation is physiologically impossible in life, and the higher the degree of relaxation the less efficiently would a 'sounding board' effect act. The body and legs of the piano are not its sounding board!

Doctor Marifioti has written more than one book on voice. In his book "Caruso's Method of Voice Production" he talks about Caruso's 'resonant body and bones,' and states that "This establishes, from the beginning, the physiological rule, that the voice must be placed in the mouth." He states that there are no registers in the human voice.

In the book "Vocal-Art-Science" by Frank E. Miller, A.M., M.D., the author has developed a special terminology which includes 'Coördination, correlation, equilibration, polar-

ization, orientation, synæsthetization, and orientopanæsthetization' of the voice. He puts triangles in the various parts of the body which are used in the act of phonation, or which he supposes to be so used, and discusses at length 'the well-known law of physics,' 'the law of pyramids!' In an illustration of a torso, he shows a triangular incision over the diaphragm (which is, of course, actually shaped like an inverted basin) and designates this area of the body 'the dynamic pyramid.' Here are some quotations from this book: "*Spiral Tone Ray*—There are two columns of air, one from the right, and the other from the left, of the lungs, sent up through the bronchial tubes, to the trachea. The right column is one-twelfth larger, in every way, than the left, because the right side of the lung is composed of three lobes, while the left is composed of but two—the heart taking the place of the third lobe. The vowel-producing air column is centrifugal, or polarized, to the right; the companion air-column is centripetal, and is for consonant production."

"The trachea is shaped like a spiral cone. These two columns of air forced up helically through the trachea, whirling around like the spire of a winding staircase; ever increasing in concentrating power-elements, that are practically vocal by the time they reach the cord, and this vertical force is the source of energy supplied to furnish vocal momentum, and vibrate the true vocal cord."

And again: "*Polarization*—a term applied by Vocal-Art-Science to the phenomenon of the power inherent in the tongue, which, owing to its position relative to the three vocal cavities (the Momentic, the Vocalic and Resonantic) and its possession of a positive and negative pole (i.e., the base and tip), tends to act as a concentrator, director and assimilator of vocal sound cyclones; the waves of the various units assuming fields very similar to those about an electro-magnet, and from here are projected into space, as a perfected tone; the application being more closely related to that of the electro-magnet than to the polarization of light . . . etc."

The author has quoted from a few of the more successful books on voice. He might quote further and mention Mr. W. Warren Shaw's theory of Speech Sounds *Produced by the Ingoing Air Currents*, or Mr. W. Henri Zay's idea of becoming

the 'center of a cosmos' and 'singing on the timbre' (which really means 'placing the voice in the nose, or masque'), or the thousands of other examples of unscientific statements on the subject of vocal technic.

No book has been found in which any mention has been made of the action of the vibrato,[1] the true facts about registration, and the muscular explanation of this phenomenon, or of pharyngeal resonance adjustment, etc. These three headings embrace the basic fundamentals of vocal technic.

It will be seen, then, that, the question is, indeed, a very serious one. Thousands of young men and women, starting with the highest hopes and aspirations, waste thousands of dollars every year on voice lessons from which they derive little benefit.

That the vocal teacher should be qualified in the sciences —physics, anatomy, physiology, psychology—and be thoroughly trained in musicianship, dramatic utterance and diction, is obvious.

A reformation from inside the profession itself is not to be hoped for, since the leading vocal teachers are among those who have written the books quoted, and those who have not committed themselves to paper seem to be equally uninformed scientifically. THEY will never disqualify themselves! The only hope lies in arousing the interest of serious scientists in the subject of voice and vocal training. The reformation must come from the outside and be forced upon a future generation of vocal teachers.

The author feels that, in coöperation with his colleagues— the late Dean Holmes C. Jackson, Doctor H. H. Sheldon, Doctor Lloyd B. Ham, S. S. A. Watkins, and other scientists —he has laid the foundations for a serious scientific study of the voice, and established certain definite fundamental principles.

There is indeed a great deal more research to be done. Suggested lines of research might embrace:

1. Determination of the ideal vowel frequencies for the perfectly used voice.

[1] Postdating the publication of several articles and the book "The Science of Voice" by the author, Prof. Carl Seashore and Arnold H. Wagner have written on this subject.

2. Investigation of the energy distribution in the tone when the technic is perfect: Determination of the ideal proportion of the vowel frequencies to the fundamental and to one another.

3. Further investigation of the vibrato: Exact determination of ideal pitch frequency variations in one vibrato. Frequency variations of the vibrato itself. Ideal amplitudes of vibrato for given intensities at different pitches, etc.

4. Investigation of the effect on quality of the higher frequencies—five thousand to twenty thousand—in phonated sounds. Effect of these frequencies on vowel determination.

5. Development of methods for the further investigation of the action of the laryngeal muscles.

6. Further investigation of the action of the resonance cavities and their inter-adjustments as a resonating system.

7. Further psychological research.

There are many other possibilities for scientific research on this subject which opens up a totally new field for scientific investigation.

All the serious research which has been made on voice in the past has dwelt, practically exclusively, upon the *average* conditions of phonation in the average individual. Our problem has been to establish the *ideal conditions*. On this side of the subject little or nothing has previously been accomplished.

The author would like to state that he would be quite willing to coöperate in any serious scientific research work on the subject of vocal technic in conjunction with physicists, physiologists and psychologists.

Our own researches had, perforce, to be suspended owing to the untimely death of Dean Jackson, the appointment of Mr. Watkins to a position which located his headquarters in London and also to the lack of appropriation for the more sensitive apparatus required for the accurate work which we had planned.

There is only one way in which the situation now existing in the field of voice training can be rectified. The scientific world will have to take a hand. Scientists will have to establish and formulate the laws which govern vocal technic, which laws must rest on the results of scientific research and deduction in all the different fields of learning involved.

After these laws have been reliably laid down, the scientific world will have to determine the course of training, in Physics, Anatomy, Physiology, Psychology, Music, Diction and Dramatic Utterance, essential for the one who intends to become a professional vocal teacher. Definite degrees for this profession may then be granted to those who qualify and NO ONE ELSE SHOULD BE ALLOWED TO TEACH.

Just as the physician is graduated and licensed by the state, so must the vocal teacher be qualified and licensed. The ex-singers, coaches, accompanists, and composers—who can, indeed, qualify as coaches of trained singers—and the mountebanks will then be prohibited from attempting to deal with the delicate and sensitive vocal apparatus, just as the quack doctor is prohibited from practising medicine.

When this has been accomplished, we will have hundreds instead of units of great artists and many thousands instead of tens of fine singers in the world.

It is only fitting that such a movement should come from The United States—the land of progress. America will then become the home and training ground of the great singers of the world. IT IS UP TO THE SCIENTISTS—only through their coöperation can this end be attained!

Intense Magnetic Fields and Low Temperature Research. SIR ERNEST RUTHERFORD. (*Nature*, Dec. 6, 1931.) In his presidential address before the Royal Society in December, 1930, Sir Ernest Rutherford, now Lord Rutherford, discussed the relative scientific advantages of relatively small grants for investigations and those requiring the provision of considerable money for several years. The Council of the Royal Society has decided in favor of the second type and has decided to support the researches in magnetism of Dr. P. Kapitza, who came in 1921 to the Cavendish Laboratory in Cambridge from Petrograd. The Royal Society made him a Messel professor and further offered the University of Cambridge £15,000 for the construction of a suitable laboratory for research both in magnetic fields and in low temperature phenomena. The University must furnish the site and assume the running expenses.

Beginning in 1922 Kapitza has developed a type of electromagnet that exceeds in its field strength all its predecessors. A current of about 70,000 amperes from a generator of special design was sent through the coils of the electromagnet and throughout a volume of 3 cu. cm. the field strength was about 350,000 gauss. "There appears to be no inherent difficulty why fields of the order of 1 million gauss should not be obtained." To keep the heating effect of the current from injuring the apparatus the current flowed for only .01 sec., and simultaneous oscillograph records of current strength, magnetic field and properties of the substance placed in the field were taken. Thus a study of the resistance of many substances from room temperature down to that of liquid air has been made. The reason for including a cryogenic plant in the proposed laboratory is this. At low temperatures where the motion of atoms and molecules is small magnetic effects occur with a minimum of complications.

In the issue of *Le Matin* of Paris for Thursday, January 15, 1931, the first column of the first page is given to a discussion by Charles Nordmann of the electromagnet of Kapitza. How often are scientific articles assigned to similar positions in the newspapers of the United States?

G. F. S.

ON THE DETERMINATION OF PRINCIPAL STRESSES FROM CROSSED NICOL OBSERVATIONS.

BY

R. V. BAUD,

Westinghouse Electric & Manufacturing Co.

STATEMENT OF THE PROBLEM.

Notations:

p, q = Principal Stresses.
m, n = Principal Stress Directions.
ϕ_m, ϕ_n = Angles between directions m, n and the positive direction of the x axis.
ϕ = General notation for either ϕ_m or ϕ_n.

Stressfield studies can be carried out either by means of two polarizing prisms, two Nicols for instance, or an interferometer or extensometers. Comparatively few laboratories are equipped with interferometer and suitable extensometers, however, and those active in research along such lines under such conditions are, and always will be, faced with the problem of obtaining complete information from crossed Nicol observations alone, a problem which dates back to 1815, when Brewster made the discovery of forced double refraction.

Maxwell [1] was the first to show 35 years later that the information from crossed Nicol observations is all that is required, fundamentally, for a complete stressfield analysis. Namely, in combining $(p - q)$, ϕ and boundary values from such observations with the differential equations obtained by formulating the equilibrium requirements of an element, see appendix I, and then integrating from free boundaries to points inside in accordance with the derived equations, the stresses p, q are obtained separately for all points of the structure. Consequently, as ϕ is also known for all points, the stressfield is fully determined in such a case, as is well known.

[1] Maxwell, "On the Equilibrium of Elastic Solids," *Trans. Roy. Soc. Edin.* 1850, and Collected Papers.

Maxwell, in proving his statement, derived the differential equations of an element marked off by four principal stress direction lines, Fig. 7. However, the particulars he advanced of how to proceed in the separation on basis of these equations are so complicated that they are of no practical value.

In 1914, Filon and Coker [2] indicated that the differential equations employing rectangular coördinates may be used for the same purpose. However, judging from the available literature, they never actually applied these equations.

Filon [3] revived Maxwell's curvilinear method in 1923, advancing important equations obtained on basis of geometrical considerations. Suggestions regarding the use of these equations and alternative methods based on the same fundamental equations are advanced in this paper.

The use of polar coördinates is furthermore suggested in this paper. When structures have a circular contour, polar coördinates are more suitable than rectangular coördinates. This statement also holds for the integration. If the structure rotates, then the integration using polar coördinates is more suitable than the two other methods of integration, because, in deriving the differential equations, the inertia forces due to rotation can easily be included.

From the above remarks it is seen that the method of separating p from q depends primarily upon the coördinates chosen. Since the coördinates are either polar, curvilinear or rectangular coördinates, the integration may be called the polar, curvilinear or rectangular integration. As a rule all three methods allow p and q to be evaluated independently from each other. The difference $(p - q)$ of the values of p and q thus obtained by integration must be equal to the observed $(p - q)$ values for all points. This offers a valuable check of the accuracy of observations and integration combined.

The differential equations involved in this study are derived in appendix I.

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[2] Filon and Coker, "Experimental Determination of the Distribution of Stress and Strain in Solids," British Association Report, 1914.

[3] Filon, "On the Graphical Determination of Stress from Photoelastic Observations," *Engineering*, Oct. 19, 1923, p. 511.

POLAR INTEGRATION.

Notations (See Figures 1 to 3):

FIG. 1.

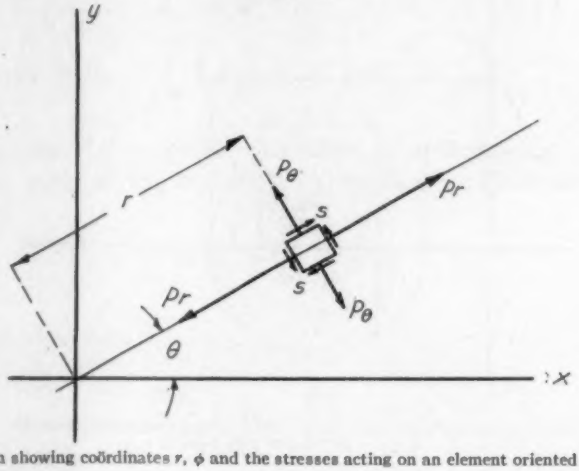


Diagram showing coördinates r, ϕ and the stresses acting on an element oriented radially.

FIG. 2.

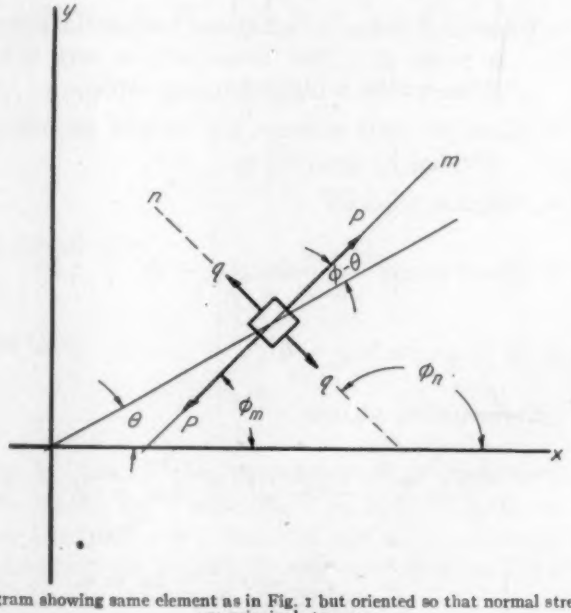
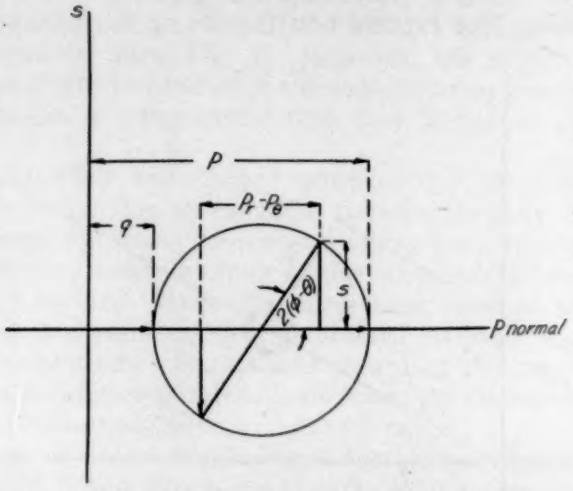


Diagram showing same element as in Fig. 1 but oriented so that normal stresses are principal stresses.

FIG. 3.



Mohr's graphical representation of stress showing relationship between s, $(p_r - p_\theta)$, p, q of Figs. 1 and 2.

p_r, p_θ = Normal stresses in radial and tangential direction.

p_{r_0}, p_{θ_0} = Normal stresses in radial and tangential directions in a point of a free boundary or any other point from which integration is started.

ρ = Mass per unit volume, viz. weight per unit volume divided by gravity g.

ω = Angular velocity.

$$s = \text{Shear stress} \frac{p - q}{2} \sin 2(\phi - \theta). \tag{1}$$

$$p_r - p_\theta = (p - q) \cos 2(\phi - \theta). \tag{2}$$

The differential equations

$$\frac{\partial p_r}{\partial r} + \frac{1}{r}\frac{\partial s}{\partial \theta} + \frac{p_r - p_\theta}{r} + \rho r \omega^2 = 0, \tag{3}$$

$$\frac{\partial p_\theta}{\partial \theta} + r\frac{\partial s}{\partial r} + 2s = 0, \tag{4}$$

can be used directly as follows:

$$p_r = p_{r_0} - \int_{r_0}^{r} \frac{1}{r}\left[\frac{\partial s}{\partial \theta} + (p_r - p_\theta) + \rho r^2\omega^2\right] dr, \qquad (5)$$

$$p_\theta = p_{\theta_0} - \int_{\theta_0}^{\theta} \left(r\frac{\partial s}{\partial r} + 2s\right) d\theta. \qquad (6)$$

Equation (5) may be obtained in a somewhat different form by multiplying equation (3) with r, and rewriting it as follows:

$$r\frac{\partial p_r}{\partial r} + p_r + \frac{\partial s}{\partial \theta} + \rho r^2\omega^2 = p_\theta,$$

which is identical to

$$\frac{\partial r p_r}{\partial r} = p_\theta - \frac{\partial s}{\partial \theta} - \rho r^2\omega^2,$$

and from this

$$p_r = \frac{r_0}{r} p_{r_0} + \frac{1}{r}\int_{r_0}^{r} \left(p_\theta - \frac{\partial s}{\partial \theta} - \rho r^2\omega^2\right) dr. \qquad (7)$$

Similarly, equation (4) multiplied with r results in

$$r\frac{dp}{\partial \theta} + r^2\frac{\partial s}{\partial r} + 2rs = 0,$$

which is identical to

$$r\frac{\partial p_\theta}{\partial \theta} = -\frac{\partial r^2 s}{\partial r},$$

and from this

$$p_\theta = p_{\theta_0} - \frac{1}{r}\int_{\theta_0}^{\theta} \frac{\partial r^2 s}{\partial r} d\theta. \qquad (8)$$

To make use of these equations it is necessary, first to insert the value for s and $(p_r - p_\theta)$ from equations (1) and (2), second to plot the integrand against r, or θ respectively, and third to find the area of the curve so obtained for successive points by any of the usual methods.

CURVILINEAR INTEGRATION.

Notations (See Figures 4, 5 and 7):

FIG. 4.

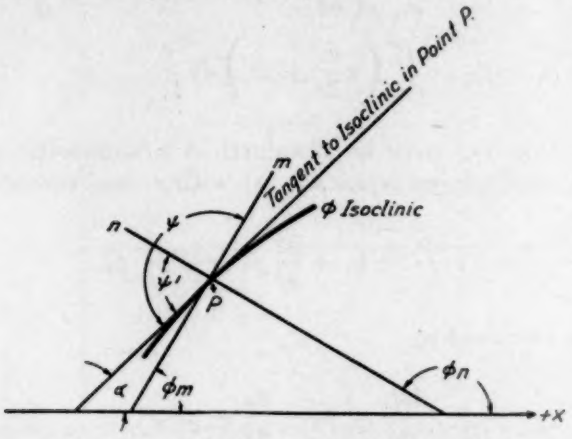
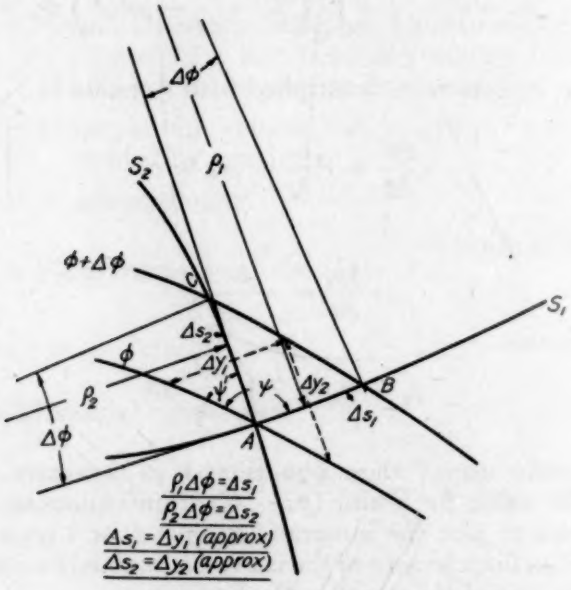


Diagram showing the angles ϕ_m, ϕ_n, α, ψ, ψ', for the case $\alpha < \phi_m < \pi/2$.

FIG. 5.



Sketch showing curvature relations.

s_1, s_2 = Orthogonal curvilinear coördinates.

p_0, q_0 = Principal stresses at a point from which integration is started.

ρ_1, ρ_2 = Radii of curvature of principal stress direction lines s_1, s_2.

ψ, ψ' = Angle through which the directions m, n have to be rotated counter-clockwise in order to make them tangent to the isoclinics.

α = Angle between the tangent to the isoclinic and the positive direction of the x axis.

$\Delta\phi$ = Constant increment in the isoclinic.

Δy_1, Δy_2 = Intercept between two near isoclinics, whose parameter differ by $\Delta\phi$ = constant; (Δy_1, Δy_2 to be measured perpendicular to the path of integration s_2, s_1).

The preparatory step in the curvilinear integration consists of constructing integral curves so that the observed directions m, n of the principal stresses p, q are tangent to the curves at all points. The curves thus obtained are called principal stress direction lines. They are used as paths for the curvilinear integration. See appendix II for definition.

The differential equations

$$\frac{\partial p}{\partial s_1} + \frac{p - q}{\rho_2} = 0,$$

$$\frac{\partial q}{\partial s_2} + \frac{p - q}{\rho_1} = 0,$$

result in the two equations (9) and (10)

$$p = p_0 - \int_{s_0}^{s_1} (p - q)\frac{1}{\rho_2} ds_1, \tag{9}$$

$$q = q_0 - \int_{s}^{s_2} (p - q)\frac{1}{\rho_1} ds_2, \tag{10}$$

upon which all curvilinear integration methods are based.

Maxwell suggested to introduce functional relations F between $(p - q)$, $1/\rho_1$ and $1/\rho_2$ and the rectangular coördinates

x, y and to compute the integrand of equations (9) and (10) for successive points along the path of integration from the F functions.

Filon's solution consists of expressing the curvatures $1/\rho_1$ and $1/\rho_2$ in terms of angles ψ and ψ' as per equations (11) and (12) *

$$\frac{1}{\rho_2} ds_1 = -\cot \psi d\phi, \tag{11}$$

$$\frac{1}{\rho_1} ds_2 = \cot \psi' d\phi, \tag{12}$$

which when inserted in equations (9) and (10) result in an integration along s_1 and s_2 within the limits ϕ_0 and ϕ.

The values of the angles ψ and ψ' are fully determined by the map of the isoclinic lines alone. To obtain the values of ψ (ψ') for the case $\alpha < \phi_m < \pi/2$, Fig. 4, and for other cases, table I may be consulted.

TABLE I.

Values of ψ, ψ'.

		ψ	ψ'
$\phi_m < \pi/2$ ($\phi_n = \phi_m + \pi/2$)	$\alpha < \phi_m$	$\pi + \delta_m$*	$\pi/2 + \delta_m$
	$\alpha > \phi_m < \phi_n$	δ_m	$\pi/2 + \delta_m$
	$\alpha > \phi_n$	δ_m	$\delta_m - \pi/2$
$\phi_n < \pi/2$ ($\phi_m = \phi_n + \pi/2$)	$\alpha < \phi_n$	$\pi/2 + \delta_n$*	$\pi + \delta_n$
	$\alpha > \phi_n < \phi_m$	$\pi/2 + \delta_n$	δ_n
	$\alpha > \phi_m$	$\delta_n - \pi/2$	δ_n
	* $\delta_m = \alpha - \phi_m$; * $\delta_n = \alpha - \phi_n$		

Isoclinic lines are observed as a rule for equal intervals $\Delta\phi$. To obtain similar lines for $\psi(\psi')$ with the same intervals $\Delta\psi(\Delta\psi')$, draw tangents to the isoclinic with the same intervals in the angle α, viz. $\Delta\phi = \Delta\alpha = \Delta\psi$ and note the point

* Eq. (11) and (12) hold for $\phi_m < \pi/2$. For $\phi_m > \pi/2$ use eq. (11*a*) and (12*a*), see appendix II.

of tangency and the values of $\psi(\psi')$ from table I for this point. By connecting points having the same angle $\psi(\psi')$ maps of lines $\psi(\psi') =$ constant are obtained, in which the paths of integration and the isochromatics may be sketched in. Such maps not only contain all the data required for the integration but they also indicate where difficulties in the integration will be experienced. Namely, for $\psi(\psi')$ equal to zero the cotangent becomes infinite but the integral itself as given by equation (9) or (10) respectively has a finite value, which cannot be computed in this case from equations (9) to (12). Incidentally, a line $\psi(\psi') = 0$ is the locus of points in which the principal stress direction lines inflect, the isoclinics being tangential to the principal stress direction lines in these points. For small angles $\psi(\psi')$ small errors in these angles produce large errors in their cotangents. Consequently, the integration through bands, say between lines $\psi(\psi') = 170°$, $0°$, $10° =$ constant becomes very inaccurate. Filon suggests to insert $1/\rho_1$ and $1/\rho_2$ from equations (13) and (14), see Fig. 5,

$$\frac{1}{\rho_2} = \frac{\Delta\phi}{\Delta y_2}, \tag{13}$$

$$\frac{1}{\rho_1} = \frac{\Delta\phi}{\Delta y_1}, \tag{14}$$

in equations (9) and (10) for the integration through such areas, whereby $\Delta\phi$ can be put before the integral.

From equations (9) and (10) it is seen that the accretions in the stress $p(q)$ depend upon the curvature of the crosswise principal stress direction line $s_2(s_1)$. In cases where simplicity is as important a consideration as accuracy, measurements made with a curvature scale will give the desired curvature values.

The method of making such curvature measurements for small portions in which equations (11) and (12) cannot be employed may represent a desirable alternative makeshift for equations (13) and (14).

A different method yet consists of plotting the values of the angles ϕ_m and ϕ_n against the coördinates s_1 and s_2 and of inserting the gradients $\partial\phi_n/\partial s_2$ and $\partial\phi_m/\partial s_1$ so obtained into

equations (9) and (10), since

$$\frac{1}{\rho_2} = \frac{\partial \phi_n}{\partial s_2}, \tag{15}$$

$$\frac{1}{\rho_1} = \frac{\partial \phi_m}{\partial s_1}. \tag{16}$$

In cases where the paths of integration have one or more inflections, preference will eventually be given to this method over the method involving the alternate use of the two sets of equations (11), (12) and (13), (14).

THE RECTANGULAR INTEGRATION.

Notations:

p_x, p_y = Normal stresses in x,y direction.

p_{x_0}, p_{y_0} = Normal stress in x,y direction in a point of a free boundary or any other point from which integration is started.

$$s = \text{Shear stress} = \frac{p - q}{2} \sin 2\phi. \tag{17}$$

The differential equations

$$\frac{\partial p_x}{\partial x} + \frac{\partial s}{\partial y} = 0, \tag{18}$$

$$\frac{\partial p_y}{\partial y} + \frac{\partial s}{\partial x} = 0, \tag{19}$$

can be used directly as follows:

$$p_x = p_{x_0} - \int_{x_0}^{x} \frac{\partial s}{\partial y} dx, \tag{20}$$

$$p_y = p_{y_0} - \int_{y_0}^{y} \frac{\partial s}{\partial x} dy, \tag{21}$$

whereby s is of the magnitude as given by equation (17).

The writer has used quotations (20) and (21) numerous times, always with satisfactory results.

SUMMARY.

1. Stressfield studies can be carried out either by means of two polarizing prisms, or an interferometer, or extensometers.

2. Crossed Nicol or similar sets are more readily available than either interferometer or suitable extensometers.

3. In case of rotation crossed Nicol observations can be made without serious difficulties, whereas interferometer or strain measurements, if possible at all, will be complicated to perform.

4. Information from crossed Nicol observations is all that is required, fundamentally, for a complete stressfield analysis.

5. The integration necessary to separate p from q from crossed Nicol observations can be carried out in various different manners, depending primarily upon the coördinates chosen to express the equilibrium requirements of an element.

6. In choosing the integration method, due considerations should be given to the shape of the structure, the forces applied to it, the simplicity in the integration and the desired accuracy of the final results.

7. Insofar as accuracy and simplicity is concerned, it is important to note that in the equations for the integration either first derivatives appear, or curvatures, or angles or intercepts between near isoclinics. It is not established as yet just how much more accurate or how much simpler any one of the methods is compared with the others. Until comparative data of some sort are available the final choice is left to the judgment of the investigator.

8. The difference $(p - q)$ of the values p, q obtained by integration must be equal to the observed $(p - q)$ values for all points. This offers a valuable check of the accuracy of observations and integration combined.

APPENDIX I.

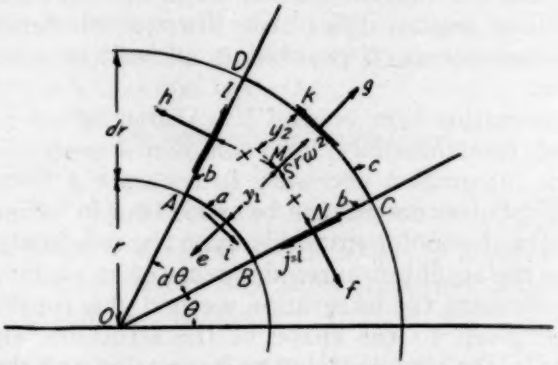
The differential equations used to separate p from q from $(p - q)$ values are derived in this appendix for the purpose of making the contents of the paper complete.

POLAR COÖRDINATES r, θ.

Each element such as $A\ B\ C\ D$ in Fig. 6 is in the state of equilibrium. The stresses acting on the sides:

$$\begin{aligned} a &= r d\theta, \\ b &= dr, \\ c &= (r + dr) d\theta, \end{aligned} \tag{1}$$

FIG. 6.



Equilibrium of an element bounded by two concentric circles and two radial lines.

of the element of unit thickness will not be perpendicular to the sides as a rule, but inclined. Such stresses are resolved into two components, namely, in components perpendicular and parallel to the sides of the element. In this manner the following stresses are obtained:

$$\left.\begin{aligned} e &= p_r, \\ f &= p_\theta, \\ g &= p_r + \frac{\partial p_r}{\partial r} dr, \\ h &= p_\theta + \frac{\partial p_\theta}{\partial \theta} d\theta, \\ i &= s_r, \\ j &= s_\theta, \\ k &= s_r + \frac{\partial s_r}{\partial r} dr, \\ l &= s_\theta + \frac{\partial s_\theta}{\partial \theta} d\theta. \end{aligned}\right\} \tag{2}$$

In addition to these stresses we have due to rotation the force $\rho r\omega^2$ per unit volume. The volume of the element is

$$b\left(\frac{a+c}{2}\right) = dr\left(rd\theta + \frac{drd\theta}{2}\right) = rd\theta\, dr,$$

as we reject quantities of the second order, such as dr^2, and further on also use the well known approximations for small angles.

Referring to Fig. 6 it is seen that

$$\left.\begin{aligned} x &= ON\tan\frac{d\theta}{2} = \left(r+\frac{dr}{2}\right)\frac{d\theta}{2}, \\ y_1 &= OM - r = \frac{r+\frac{dr}{2}}{\cos\frac{d\theta}{2}} - r = \frac{dr}{2}, \\ y_2 &= \frac{dr}{2} \end{aligned}\right\} \qquad (3)$$

and we obtain from equilibrium requirements

(*a*) *For the moments around point M:*

$$(ia + kc)y_1 - (j + y)bx = 0,$$

or explicitly, inserting the values from equations (1), (2), and (3)

$$s_r\frac{dr}{2}rd\theta + \left(s_r + \frac{\partial s_r}{\partial r}dr\right)\frac{dr}{2}(r+dr)d\theta - s_\theta\left(r+\frac{dr}{2}\right)\frac{d\theta}{2}dr - \left(s_\theta + \frac{\partial s_\theta}{\partial\theta}d\theta\right)\left(r+\frac{dr}{2}\right)\frac{d\theta}{2}dr = 0,$$

and from this

$$2s_r r\frac{dr}{2}d\theta - 2s_\theta r\frac{d\theta}{2}dr = 0,$$

$$s_r = s_\theta.$$

(*b*) *For the forces acting radially* (+ *in direction outwards*)

$$gc - ea + (f+h)b\sin\frac{d\theta}{2} + (l-i)b\cos\frac{d\theta}{2} + \rho r^2\omega^2 d\theta dr = 0,$$

or explicitly,

$$\left(p_r + \frac{\partial p_r}{\partial r}dr\right)(r + dr)d\theta - p_r r d\theta - \left(p_\theta + p_\theta + \frac{\partial p_\theta}{\partial \theta}d\theta\right)dr\frac{d\theta}{2} + \frac{\partial s}{\partial \theta}d\theta dr + \rho r^2\omega^2 d\theta dr = 0,$$

and from this

$$\frac{\partial p_r}{\partial r} + \frac{1}{r}\frac{\partial s}{\partial \theta} + \frac{p_r - p_\theta}{r} + \rho r\omega^2 = 0.$$

(*c*) *For the forces acting tangentially* (+ in counter-clockwise direction)

$$(-f + h)b\cos\frac{d\theta}{2} + (i + l)b\sin\frac{d\theta}{2} - ia + kc = 0,$$

or explicitly,

$$\left(-p_\theta + p_\theta + \frac{\partial p_\theta}{\partial \theta}d\theta\right)dr + \left(s + s + \frac{\partial s}{\partial \theta}d\theta\right)dr\frac{d\theta}{2} - s r d\theta + (r + dr)d\theta\left(s + \frac{ds}{\partial r}dr\right) = 0$$

and from this

$$\frac{\partial p_\theta}{\partial \theta} + r\frac{\partial s}{\partial r} + 2s = 0.$$

ORTHOGONAL CURVILINEAR COÖRDINATES S_1, S_2.

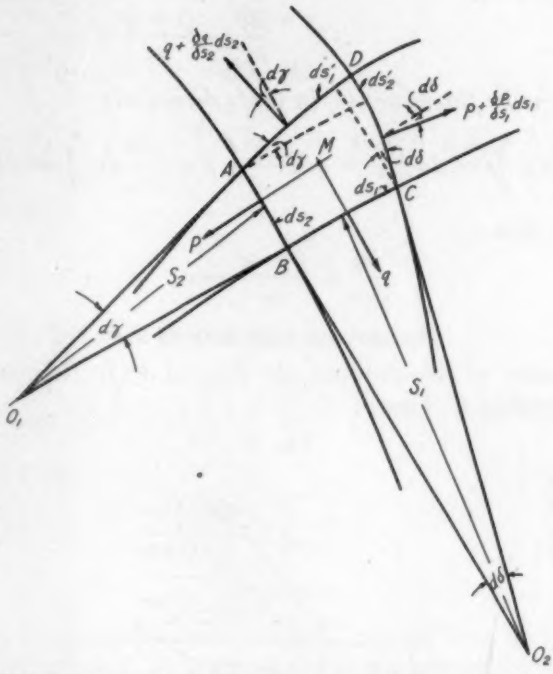
From Fig. 7 it is seen that the sides ds_1, ds_2, ds_1^1, and ds_2^1 of the element *ABCD* of unit thickness are related to each other as is indicated by eqs. (4) and (5)

$$ds_1^1 = ds_1 + ds_2 d\delta = ds_1 + ds_2\frac{ds_1}{\rho_1}, \tag{4}$$

$$ds_2^1 = ds_2 + ds_1 d\gamma = ds_2 + ds_1\frac{ds_2}{\rho_2}. \tag{5}$$

The stresses are principal stresses by definition, viz., there are no shear stresses acting along the sides of the element.

FIG. 7.



Equilibrium of an element bounded by four near principal stress direction lines.

Referring to Fig. 7 we obtain from equilibrium requirements

(*a*) *For the forces acting in OM_1 direction:*

$$\left(p+\frac{\partial p}{\partial s_1}ds_1\right)\cos d\delta ds_2^1 - pds_2 - \left(q+\frac{\partial q}{\partial s_2}ds_2\right)\sin d\gamma ds_1^1 = 0.$$

Inserting the values for s_1^1 and s_2^1 from equations (4) and (5), we obtain

$$\left(p+\frac{\partial p}{\partial s_1}ds_1\right)\left(ds_2+ds_1\frac{ds_2}{\rho_2}\right)-pds_2$$
$$-\left(q+\frac{\partial q}{\partial s_2}ds_2\right)\frac{ds_2}{\rho_2}\left(ds_1+ds_2\frac{ds_1}{\rho_1}\right)=0,$$

$$pds_1\frac{ds_2}{\rho_2}+\frac{\partial p}{\partial s_1}ds_1ds_2-q\frac{ds_2}{\rho_2}ds_1=0,$$

and from this

$$\frac{\partial p}{\partial s_1} + \frac{p - q}{\rho_2} = 0.$$

(*b*) *For the forces acting in OM_2 direction:*

$$\left(q + \frac{\partial q}{\partial s_2} ds_2\right) \cos d\gamma ds_1{}^1 - q ds_1 - \left(p + \frac{\partial p}{\partial s_1} ds_1\right) \sin d\delta ds_2{}^1 = 0$$

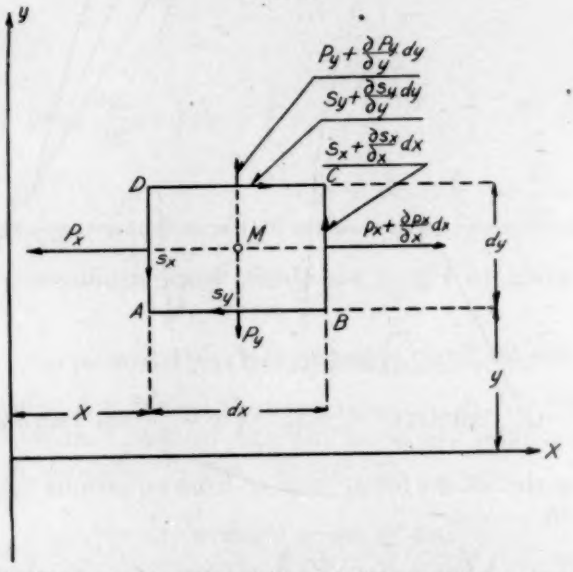
and from this

$$\frac{\partial q}{\partial s_2} + \frac{p - q}{\rho_1} = 0.$$

RECTANGULAR COÖRDINATES *x, y.*

The sides of the element are dx and dy in this case. We have, referring to Fig. 8,

FIG. 8.



Equilibrium of an element bounded by straight lines parallel to *x, y* axes.

(*a*) *For the moments around point M:*

$$\left[s_y + \left(s_y + \frac{\partial s_y}{\partial y} dy\right)\right] dx \frac{dy}{2} - \left[s_x + \left(s_x + \frac{\partial s_x}{\partial x}\right) dx\right] dy \frac{dx}{2} = 0,$$

$$2 s_y dx \frac{dy}{2} - 2 s_x dy \frac{dx}{2} = 0,$$

and from this

$$s_x = s_y.$$

(*b*) *For the forces in x-direction:*

$$\left(p + \frac{\partial p_x}{\partial x} dx\right) dy + \left(s + \frac{ds}{\partial y} dy\right) dx - p_x dy - s\, dx = 0$$

and from this

$$\frac{\partial p_x}{\partial x} + \frac{\partial s}{\partial y} = 0.$$

(*c*) *For the forces in y-direction:*

$$\left(p_y + \frac{\partial p_y}{\partial y} dy\right) dx + \left(s + \frac{\partial s}{\partial x} dx\right) dy - p_y dx - s dy = 0$$

and from this

$$\frac{\partial p_y}{\partial y} + \frac{\partial s}{\partial x} = 0.$$

APPENDIX II.

In deriving equations (11) and (12), Filon defined ϕ as that angle, which d_{s_1} or p respectively makes with the positive direction of the axis of x. Since in the experiment ϕ is varied from zero to ninety degrees, it follows from this definition that the direction of p is restricted to the first (and third) quadrant, and the direction of q is limited to the second (and fourth) quadrant. Consequently, when this definition for p and q is accepted, then a line such as shown in Fig. 9 is a p principal stress direction line for the length BC; equations (9) and (11) have then to be employed in computing p; the remainder of the line AD is a q principal stress direction line, and in integrating from A to B and from C to D, equations (10) and (12) have to be used. It is seen that in points B and C the stress q changes to p and vice versa. The above definition for p and q therefore not only lacks precision for these points, but the stress field as a whole is not defined satisfactorily in this manner.

The method that has been adopted by the writer consists of denoting arbitrarily one of the principal stress direction

lines, such as shown in Fig. 9, a p line. All lines intersecting this p line are then q lines and in turn all lines intersecting q lines are p lines. This defines the stress field more satisfactorily.

FIG. 9.

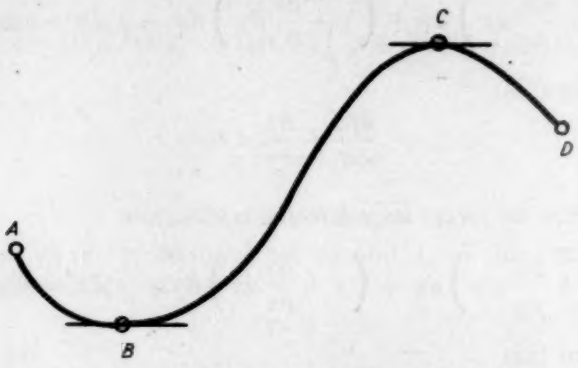


Illustration of definition of principal stress direction line.

In order to account for this change in definition, it is necessary only, to replace equations (11) and (12) by equations (11a) and (12a)

$$\frac{1}{\rho_2} ds_1 = \quad \cot \psi' d\phi, \tag{11a}$$

$$\frac{1}{\rho_1} ds_2 = - \cot \psi d\phi, \tag{12a}$$

provided $\phi_m > \pi/2$. For points in which $\phi_m < \pi/2$, equations (11) and (12) should be used as given in the paper.

To illustrate this, assume first that the line AD in Fig. 9 is a p principal stress direction line; equation (9) is used in this case. From A to B and from C to D insert the value from equation (11a), since $\phi_m > \pi/2$ for these parts of the p line. From B to C insert the value of equation (11) since for this part $\phi_m < \pi/2$.

The same numerical results are obtained if AD is defined as a q principal stress direction line; equation (10) is used in this case. From A to B and from C to D insert the value of equation (12), since $\phi_m < \pi/2$ for these parts of the curve. From B to C, $\phi_m > \pi/2$ and therefore insert the value of equation (12a).

A SOLUTION FOR THE GEOMETRICAL TRISECTION OF ANGLES AND THE PROPORTIONAL DIVIDING OF ARCS.

BY

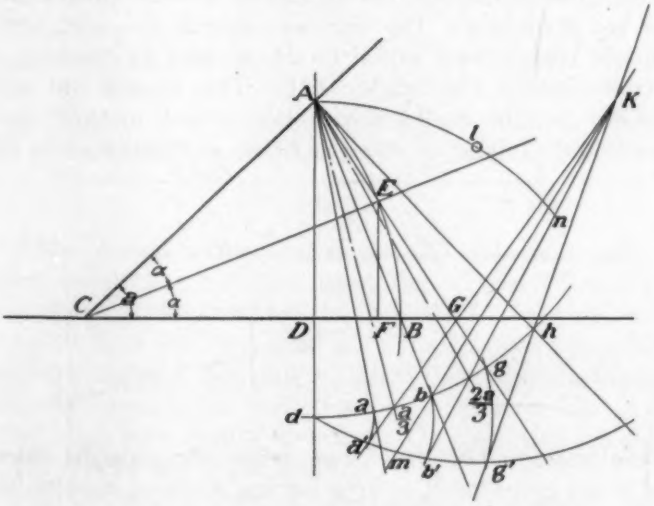
L. J. R. HOLST,

Member of the Institute.

In the course of some geometrical studies the writer found that when a line is drawn uniting the upper ends of the sine of an angle and the sign of half that angle, the line so drawn makes with either of these sines an angle three fourths as large as the original angle.

In other words, when angle $ACB = a$ (Fig. 1) is bisected and the sines AD of a and EF of $a/2$ are drawn, the angle $DAG = \frac{3}{4}ACB$.

FIG. 1.



The proof herefor is readily given by noting that the triangle ACE is isosceles by construction, and hence that the

angle CAE is $90 - \alpha/2$. Angle CAD is $90 - 2\alpha$ and consequently:

$$DAG = CAE - CAD = 90 - \alpha/2 - (90 - 2\alpha) = 1\tfrac{1}{2}\alpha = \tfrac{3}{4}a.$$

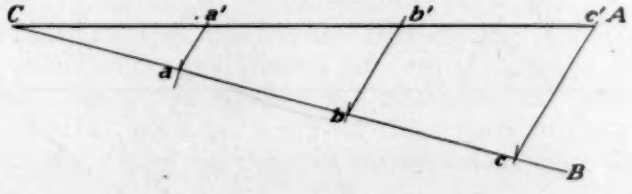
The occurrence of an angle bearing this relation to another angle gave rise to the suggestion that some further study might lead to the solution of the problem of trisecting an angle and it is thus of interest to note that when drawing the chord AB of the original angle a, this chord will exactly trisect the angle DAG.

The proof of this is equally simple. Since the triangle ACB is isosceles by construction, its angle CBA is $90 - \alpha$ and as the angle DGA is $90 - 1\frac{1}{2}\alpha$, the angle BAG, which is their difference, is $\frac{1}{2}\alpha$.

However this is one fourth of the original angle ACB and the problem is thus not yet solved, notwithstanding that the angle DAG has been trisected by purely geometrical means.

If now the arc d–a–b–g be lengthened out to h, so that gh equals bg, the angle dAh will equal ACB, and I will proceed to show how the arc dh can be divided proportionally to an arc d–a'–b'–g'. On this arc chords da', $a'b'$, and $b'g'$ are made respectively equal to da, ab and bg resulting from the trisecting of the angle DAG: The means are strictly analogous to the well-known geometrical method for the proportional division of straight lines, as illustrated in Fig. 2.

FIG. 2.

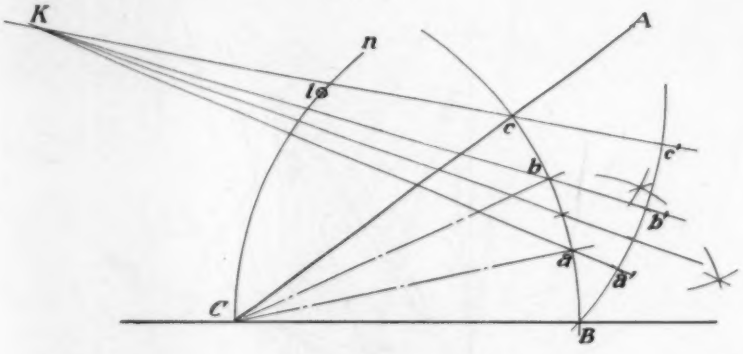


This analogy becomes clear, when the straight lines CA and CB are considered as arcs having radii of infinite length and when the parallel lines $a'a$, $b'b$ and $c'c$ are considered as radii of infinite length, swinging around an infinitely distant center in the direction $c'c$.

The application to arcs of finite curvature can be made by first selecting on the arc *Aln* (Fig. 1) drawn with *dA* as radius from *d* as center, to serve an arbitary point *l* as center for the arc *da'b'g'*, which has thus a radius *ld* = *Ad*. On this arc the chords *da'* = *da*, *db'* = *db*, and *dg'* = *dg* are now set off, and a line *g'h* is drawn and sufficiently produced upwardly. This is the equivalent for *cc'* in Fig. 2, which determines the proportion. Next the arc *dg'* is bisected in *m* and the line *mb* is drawn until it intersects *g'h* in *K* (*b* being the middle of the arc *dh* by construction). *K* will then be the center of origin of lines *Kg'*, *Kb'* and *Ka'* which respectively cut the arc *dh* in points which trisect it.

Having now derived a method for the trisecting of an angle, I will show how it may be applied to an angle in situ, with the aid of Fig. 3.

FIG. 3.



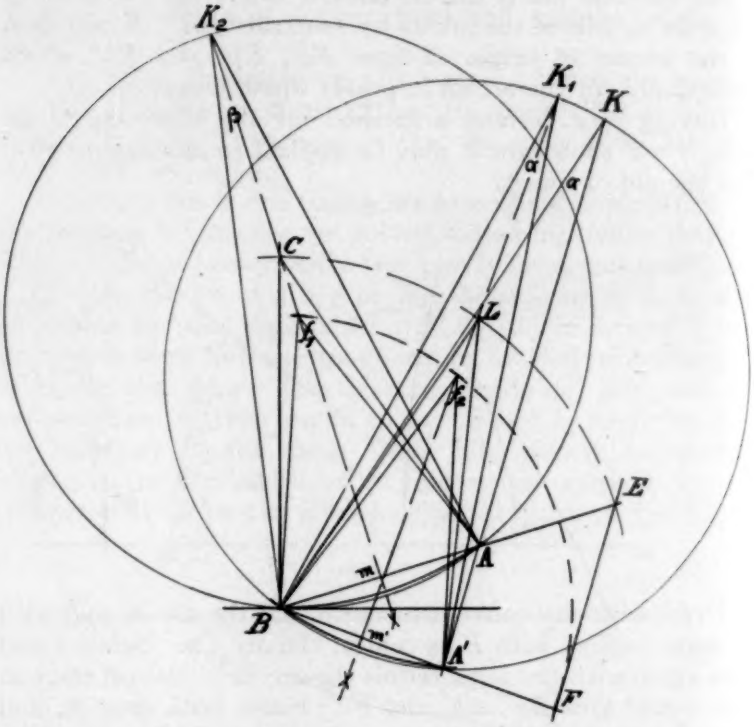
Draw with any convenient radius *CB* the arc *Bc* and with the same radius, with *B* as center, the arc *Cn*. Select *l* and draw again with the same radius the arc *Bc'*. Set off thereon three equal arcs *Ba'*, *a'b'* and *b'c'*; bisect both arcs *Bc* and *Bc'* and determine *K* by drawing a line through these bisecting points and through the end points of both arcs. Next draw *Kb'* and *Ka'* and finally the radii *Ca* and *Cb*, which will correctly trisect the angle *ACB*.

It will be noted that in this construction there was no previously determined length for the arcs *Ba'*, *a'b'* and *b'c'*, the only condition being that they be equal. This is because any line radiating from *K* will cut proportional sections

off either arc. The proof for this theorem yields incidentally a more accurate means to determine the point K.

In Fig. 4 the arcs BA and BA' have been constructed as before, and the chords drawn in. The chord BA has been extended to E, so as to make $BE = BC$. The line $A'A$ is drawn as usual and EF is drawn parallel to the latter, so that $BE : BF = BA : BA'$. Also the bisectors Cm and Lm'

FIG. 4.



have been drawn in their respective angles. Now set off on these bisectors the points f_1 and f_2 so that Bf_1 and Bf_2 equal BF, and from f_2 as center draw a circle with the radius Bf_2. This circle cuts $A'A$ in K and $m'L$ in K_1. The angles BKA' and BK_1A' stand in this circle on the same chord BA' and are thus equal. Further draw the angle Bf_2A'. Since this angle and the angle BLA' stand on the same arcs, they are

related inversely to their radii, which bear a constant ratio. If now also a circle be struck with the radius Bf_1 from the center f_1, we find that the corresponding angles constructed on the arc BA bear the same constant ratio to each other. The same relations exist for any other pair of chords, cut from the arcs BA and BA' by any line radiating from K within the limits of either total arc, and their relation remains constantly as that of the radii BE and BF, which by construction is as BA to BA', and they are thus proportionally divided by any line passing through K.

Panes of Quartz. (*Journal of the Optical Society of America*, Jan., 1930.) Elihu Thomson some years ago set about the preparation of a transparent medium that, unlike glass, should be transparent to ultra-violet radiation. Fused quartz was the best material but its use was fraught with many difficulties and their conquest required years of experiment. When quartz melts at about 3200° F. the resulting liquid is very viscous, in marked contrast to the low melting point of glass and to its mobility.

Quartz crystal is crushed to one-quarter inch mesh and melted in a vacuum on thin graphite slabs by means of an electric furnace. Nitrogen under 150 lbs. pressure is later admitted to the fused material for the purpose of compressing the bubbles. The quartz plates thus produced need no polishing, cutting or grinding except at the edges. The price is such that solaria can be furnished with them in place of glass. One solarium thus outfitted was dedicated at Saranac Lake by the National Vaudeville Artists.

G. F. S.

X-RAY STUDIES OF MOTIONS OF MOLECULES IN DIELECTRICS UNDER ELECTRIC STRESS.

BY

RALPH D. BENNETT, Ph.D.,

University of Chicago.

Among the earliest manifestations of molecular motion in dielectrics was the effect discovered by Kerr, that a dielectric becomes optically anisotropic when subjected to electric stress. This effect can be explained by assuming that the molecules of the dielectric are either permanently polarized or become so by induction. The electric stress acting on these molecules produces orientation which results in double refraction.

The electromagnetic theory predicts that the square of the index of refraction should equal the dielectric constant. This is in general not true. The discrepancy can be explained if we assume that molecules are rotated by the electric field. At low frequencies where the dielectric constant is usually measured the polar molecules have time to orient themselves. The polarization due to orientation added to the induced polarization produces a relatively large value. At optical frequencies where the index of refraction is measured the orientation is unable to follow the rapid alternations of field strength, resulting in a lower value for the dielectric constant.

If fields of frequencies between these extreme limits are applied starting at low frequency and increasing, a region will be found where a transition takes place from a higher to a lower value of dielectric constant. In this same region the dielectric absorption passes a maximum, indicating that at this frequency a maximum amount of rotation takes place. The value of the critical frequency depends on the temperature and can be varied in the case of rosin from audio frequencies to radio frequencies by a rather small rise in temperature.

Three experiments have been performed recently in the Ryerson Physical Laboratory, attempting to measure molecular motion by means of X-rays. The first [1] was an attempt

[1] R. D. Bennett, *Phys. Rev.*, **36**, p. 65, 1930.

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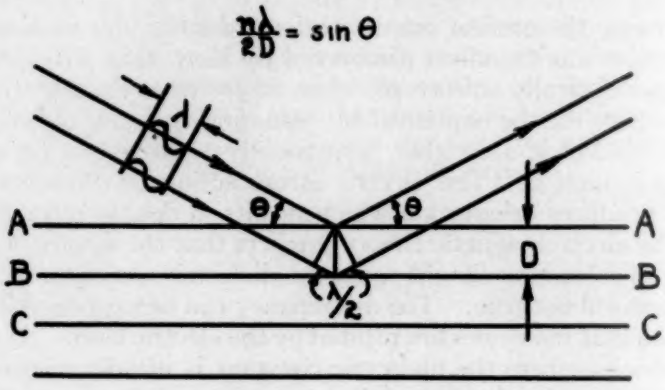
[1] R. D. Bennett, *Phys. Rev.*, 36, p. 65, 1930.

to detect molecular motions in a solid dielectric. The method can best be shown by Fig. 1. The lines *AA*, *BB*, *CC* represent planes of atoms in a crystal lattice. The incident x-rays are scattered in all directions by the electrons in the crystal, but for a given wave-length there will be angles θ where constructive interference takes place. The intensity of the reflection in such a case can be shown to depend on the distribution of

FIG. 1.



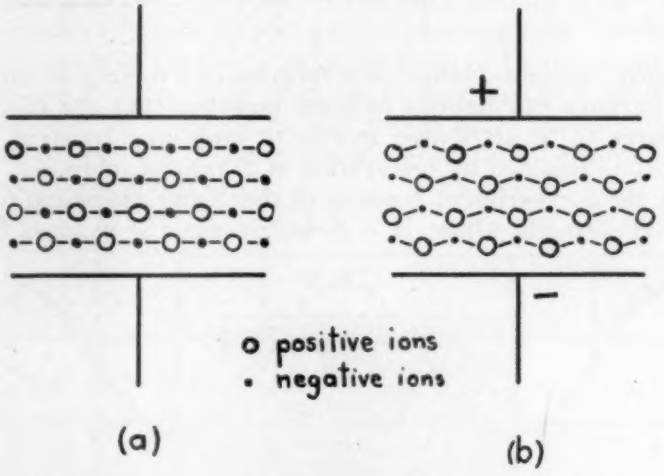
the electrons about the geometric planes *AA*, *BB*, *CC*. If now we choose a polar lattice such as NaCl, or KCl, represented in Fig. 2(*a*) where alternate atoms are charged as indicated, and subject it to electric stress, the atoms of the lattice might be expected to be displaced (Fig. 2(*b*)). This amounts to roughening the reflecting planes and reduces the reflecting power. The existence of a significant reduction in reflecting power is doubtful, according to the experimental results of the writer.

J. Hengstenberg [2] claims to have found a small effect of the order of 1/10 per cent. and proposes a theory to explain it quantitatively. His theory attributes the whole polarization to motion of the ions as units. It is hard to see how this theory can be entirely correct since as a result of it we should expect a non-polar lattice such as diamond to have a dielectric constant of unity instead of the observed value of 16.5. The writer suggests that the discrepancy in experimental re-

[2] J. Hengstenberg, *Zs. für Phys.*, **58**, p. 345, 1929.

sults may lie in the method of taking observations. Hengstenberg's method consisted of direct readings of the ionization current—a quantity which would be very difficult to keep steady to 1/10 per cent. The writer's method consisted of averaging this quantity over a large number of periods with the stress alternately on and off. Such a method would tend to eliminate the effect of fluctations.

FIG. 2.



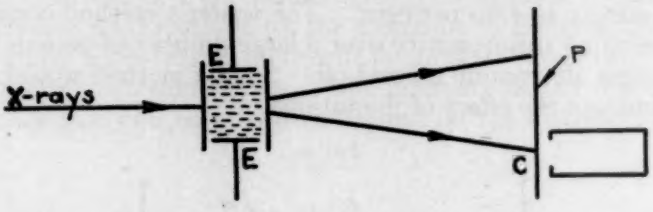
The result to date indicates that the dielectric polarization in this type of solid may be due partly to distortion of the lattice, and partly to distortion of the atoms which make it up.

The second experiment is that of MacFarlan [3] on the scattering power of a liquid under electric stress. If a narrow beam of X-rays is passed through a liquid to a photographic plate as in Fig. 3 there results a diffraction halo. The intensity distribution indicates that there must be some sort of orderly grouping of the molecules, and the size of the halo gives a measure of their average closeness of approach. Suppose a potential is applied between the electrodes *EE* and the intensity at *C* measured first with the field off and then with it on. MacFarlan found an increase of about 2 per cent. with the application of the field for highly polar nitrobenzene

[3] R. L. MacFarlan, *Phys. Rev.*, **35**, p. 1469, 1930.

while in non-polar benzene he found none. This change in

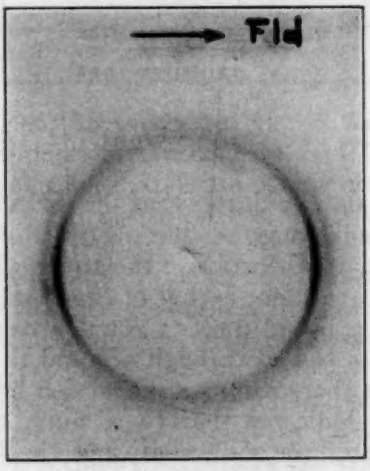
FIG. 3.



intensity can be explained as a rotation of a molecular nature. MacFarlan's calculations indicate however that the effect is too large to be attributed merely to molecular rotation, but could be explained by orientation of groups of molecules.

A third experiment consists in the X-ray investigation of motions brought about in a dielectric when it changes from

FIG. 4.



Halo from paraffin solidified under electric stress.

liquid to solid under electric stress. Ewing [4] has shown that a mixture of waxes under these conditions shows anisotropic scattering power for X-rays. The writer has made similar

[4] Maurice Ewing, *Phys. Rev.*, **36**, p. 378, 1930 (*A*).

experiments on paraffin. Ordinary paraffin consists of a mixture of a number of hydrocarbons of the general formula C_nH_{2n+2}. The material used had a fairly definite melting point and X-ray measurements indicate that it consisted chiefly of molecules $C_{27}H_{56}$. Fig. 4 shows a diffraction halo from paraffin which has solidified under electric stress. The halo is not that of a liquid, but is typical of a powdered crystal aggregate in which the individual crystals are very small. There is a marked increase in intensity for the rings due to side spacings (3.8 A. U. and 4.2 A. U.) along a line parallel to the direction of the field, and a decrease in the direction perpendicular to this. The rings due to long spacings (37 A. U.) fall so near the main beam that it was not possible with the apparatus at hand to make a good photographic record of the variation in intensity.

FIG. 5.

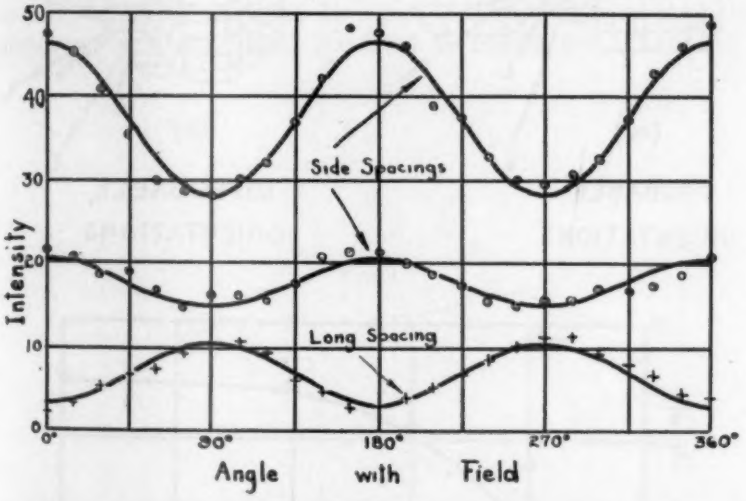


Fig. 5 shows the result of ionization measurements on the two side and one long spacing rings. The intensities follow sinusoidal variations, that of the long spacing being 180° out of phase with those of the side spacings.

The hydrocarbons are known to crystallize in thin flakes with the long chains perpendicular to the face of greatest area.[5] If these flakes are assumed to orient themselves

[5] Müller, *Proc. Roy. Soc.*, *A* 120, p. 437, 1928.

edgewise to the field the observed intensity distribution can be accounted for. In Fig. 6 (*a*) are shown orientations of the crystals edgewise to the field with the resulting intensities indicated. Fig. 6 (*b*) shows orientations perpendicular to the

FIG. 6.

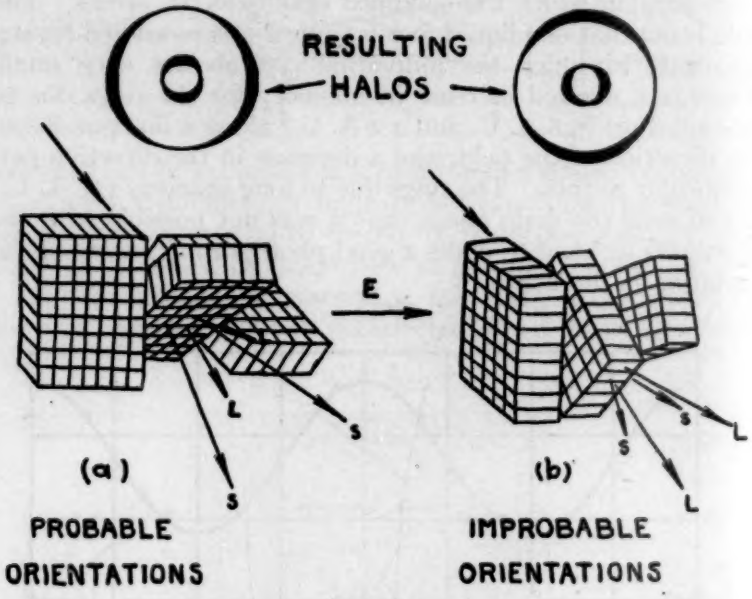
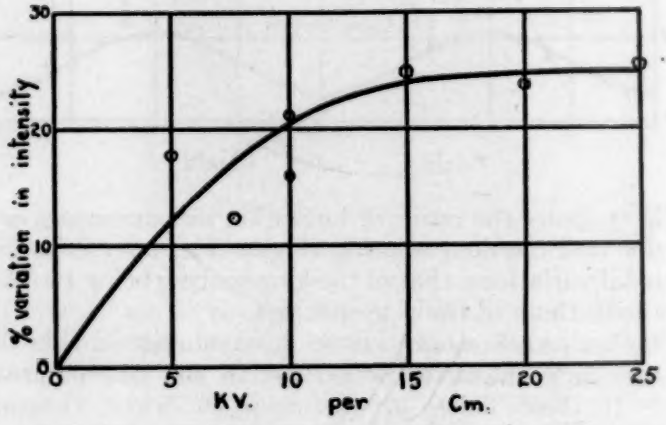


FIG. 7.



field with the resulting intensities. The measurements indicate that crystals oriented as in Fig. 6 (*a*) are frequent while those oriented as in Fig. 6 (*b*) are rare.

Figure 7 shows the relation between variation of reflecting power and gradient, or indirectly the relation between orientation and gradient. The curve is a typical saturation curve indicating that ordinary gradients are sufficient to align most of the small crystals.

This experiment leads to the conclusion that the molecular group state of the liquid is accentuated as the liquid solidifies leading to definite crystalline structure, which can be strongly oriented by the electric field. We may expect to find that such an oriented crystalline structure exhibits different dielectric properties in different directions.

These experiments indicate that X-ray methods may prove a useful tool in the investigation of molecular motions in dielectrics. The work at Ryerson Laboratory was in part supported by the Utilities Research Commission.

Constitution of Tungsten. F. W. ASTON. (*Nature*, Dec. 13, 1930.) After many failures Aston has at last succeeded in getting the mass-spectrum of tungsten. It was the preparation of the volatile carbonyl, $W(CO)_6$, by A. v. Grosse of Berlin that made this result possible. The carbonyls of chromium and molybdenum had rendered the same service in the case of these metals.

Tungsten was found to have four isotopes, the mass numbers and relative abundance of which are as follows:

Mass Number	182	183	184	186
Percenage Abundance	22.6	17.2	30.1	30.0

The atomic weight is calculated from these data to be 183.96, which agrees well with the common value 184.

G. F. S.

DETERMINATIONS ON THE SIZE-FREQUENCY DISTRIBUTION OF RESIDUAL (UNDEVELOPABLE) GRAINS OF A PHOTOGRAPHIC EMULSION.*

BY

E. C. JENSEN and A. P. H. TRIVELLI.

Introduction.—From a consideration of data already on hand representing the size-frequency distribution of the residual grains, for different times of exposure, of a certain photographic emulsion, we believed it possible, by applying some frequency formula to the series of distributions, to determine if some simple relation exists between exposure and the size-frequency of residual grains. (Grains left after the removal of exposed and developed grains are termed residual grains.) The emulsion used was at the time the best available for obtaining this kind of data. Although not originally intended for this particular study it is believed suitable for a preliminary investigation.

The relation of the size-frequency distribution of the grains of silver halide in a photographic emulsion and exposure has never been successfully formulated. Certainly the confirmation with statistical size-frequency data of any theoretical exposure formula has not met with any degree of success. Silberstein's original quantum theory formula of exposure and also its modified form [1] were not upheld by any experimental findings. The theory appeared at first to represent roughly the observed facts as found by Trivelli and Righter [2] using a great range of sizes of large clumps of grains, but later investigations showed that it failed to represent distributions of the grains of the smaller class sizes, and F. C. Toy's criticism [3] was found to be materially correct by Sheppard, Trivelli and Loveland.[4] Wightman, Trivelli and Sheppard [5] in their study of

* Communication no. 458 from the Kodak Research Laboratories.

[1] *Phil. Mag.*, 44, 257, 1922.

[2] *Phil. Mag.*, 44, 252, 1922.

[3] *Phil. Mag.*, 45, 715, 1923.

[4] J. FRANK. INST., 200, 51, 1925.

[5] *J. Phys. Soc.* 28, 529, 1924.

the relation of size-frequency distribution to sensitometric characteristics, worked out a method for determining experimentally the densities for any given exposure in the characteristic curve of a simple photographic emulsion from its grain size-frequency distribution but found poor agreement between observed and calculated values.

In both the cases just mentioned a theoretical formula has been set up and an attempt made to fit it with the data. Mr. R. P. Loveland of these Laboratories has suggested that a study and analysis, graphical and mathematical, of the complete size-frequency distribution of the residual (undevelopable) grains for a number of different exposure times would be of value in determining the relation of exposure and number and size of grains made developable, possibly of formulating it, and has pointed out that if some frequency formula could be found to represent the whole series of distributions its parameters would be very useful as constants for this study. Loveland and Trivelli [6] developed methods for the analysis of size of particles which they applied to the frequency distributions of a number of photographic emulsions. Some of these same methods will be used in the brief analysis here undertaken.

TABLE I.

Size-Frequency of Residual Grains per Cm³ of Plate $\times 10^{-4}$.

Step No.	Exposure in Secs.	Mean Class Size—Area in μ^2.										
		0.115	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9	2.1
0	0	1428.9	1277.0	317.97	58.90	16.63	5.52	1.69	1.42	0.88	0.18	0.18
1	4	1347.5	1110.1	237.28	28.27	8.00	2.38	1.79	0.91			
2	5.7	1452.2	1035.0	171.92	15.42	2.08	1.16	0.30	0.58			
3	8	1392.4	882.12	98.39	8.81	1.19	0.30	0.59				
4	11.3	1278.2	621.90	47.25	3.12	0.88	0.22					
5	16	995.12	302.69	21.03	1.18	0.59						
6	22.6	685.29	152.45	10.60	0.71	0.18						
7	32	384.37	85.41	3.40								
8	45.2	245.02	54.34	1.45								
9	64	152.53	26.03	0.21								
10	90.5	100.13	8.23									
11	128	56.23	1.96									

Experimental.—The experimental data shown in Table I, have been obtained in this Laboratory and were previously

[6] J. FRANK. INST., 204, 193, 1927.

used by Wightman, Trivelli and Sheppard in the work mentioned, relative to density derivations. They are for a simple photographic emulsion and the procedure by which they were obtained is fully described by them. Included is the original grain frequency distribution and that of the residual grains for eleven different exposure steps.

It has previously been found that the original frequency distribution of this particular emulsion is fitted by neither the simple exponential equation

$$y = Ae^{-kx}$$

nor that of the Gaussian type

$$y = Ae^{-k(x-\alpha)^2},$$

but that it is well represented by the equation derived by Loveland from the general parabolic formula

$$Y = Ae^{-k(\ln x - \alpha)^2}$$

which represents the data when the logarithms of both x and y values are plotted.

The latter formula was the one first applied by us to the residual grain distributions when it was found that by plotting them on double log paper they resembled parabolas much the same as the original (unexposed) grain distribution. It was applied to several of the distributions by the method of least squares and fitted well enough so that no attempt was made at using any other formula. Figure 1 shows the parabolic double log curves for both original and residual grains, drawn with the aid of a string and triangle. The data are insufficient for the drawing of curves for any more than the first seven exposure steps. A small amount of smoothing has been done in drawing them, by means of the size-frequency curves (Fig. 2) most of which applied to the smallest or first class size.

It is apparent that these curves do not all have the same shape or curvature, *i.e.*, could not be superimposed, and this is also seen by an examination of the values calculated for the parameter k (Table II). These parameters, k, α, and A were

FIG. 1.

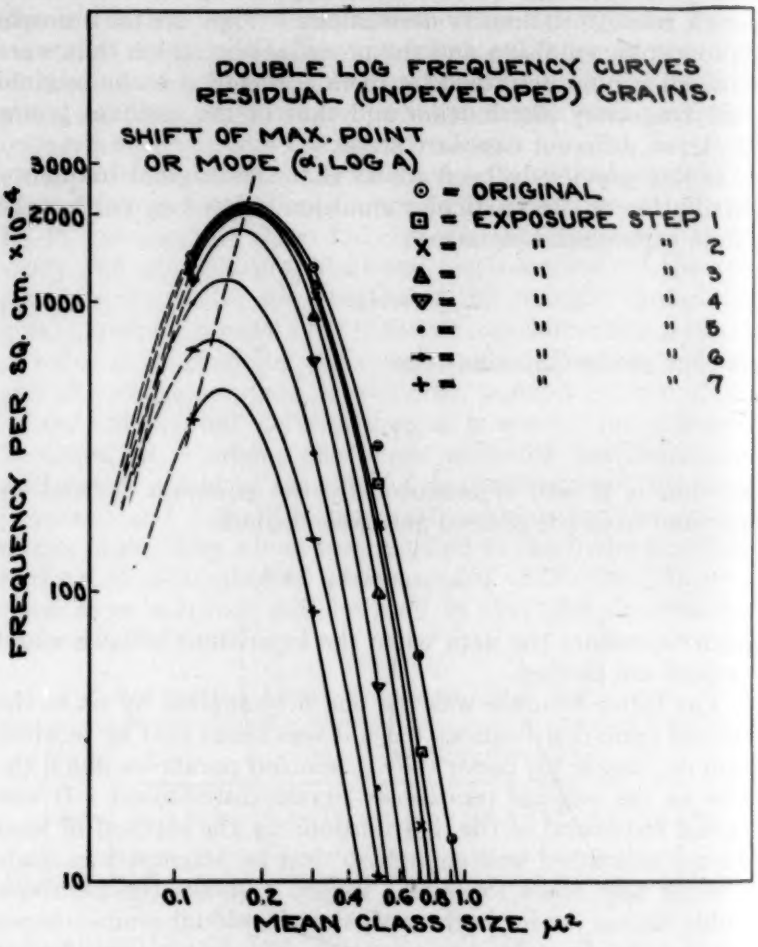


TABLE II.

Parameters of Frequency Formula $y = Ae^{-k(\ln x - a)^2}$ *Applied to Size-Frequency Distributions of Residual Grains.*

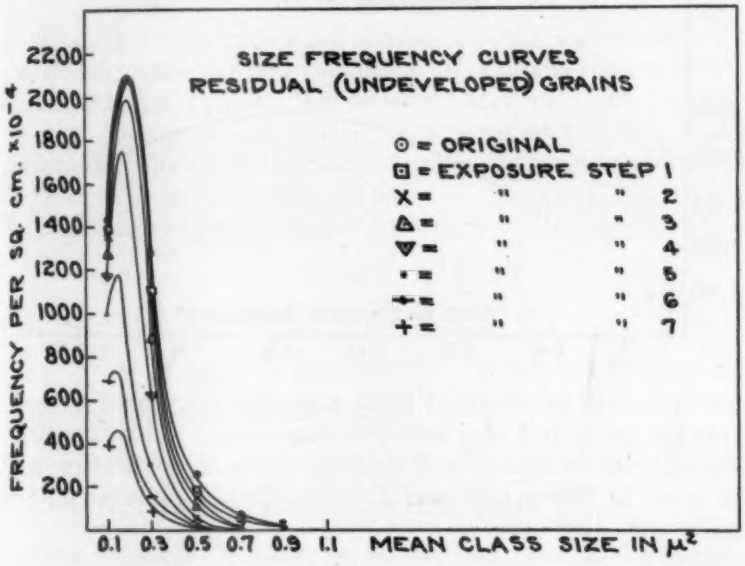
Exposure Step.	α.	k.	A.
0	− 1.715	1.804	2049.95 × 10^4
1	− 1.735	2.267	2150 "
2	− 1.736	2.595	2163 "
3	− 1.755	2.665	1984 "
4	− 1.795	2.960	1750 "
5	− 1.912	2.707	1178 "
6	− 1.998	2.484	733 "

calculated algebraically from the frequency formula applied

$$y = Ae^{-k(ln\ x - \alpha)^2}.$$

In each case three values read from the double log curves were used for the determination; k, which determines the shape of the parabola and the degree of spread of the ordinary frequency curve, is not constant but is found to increase with exposure to a maximum, and then to decrease (Fig. 3). The

FIG. 2.

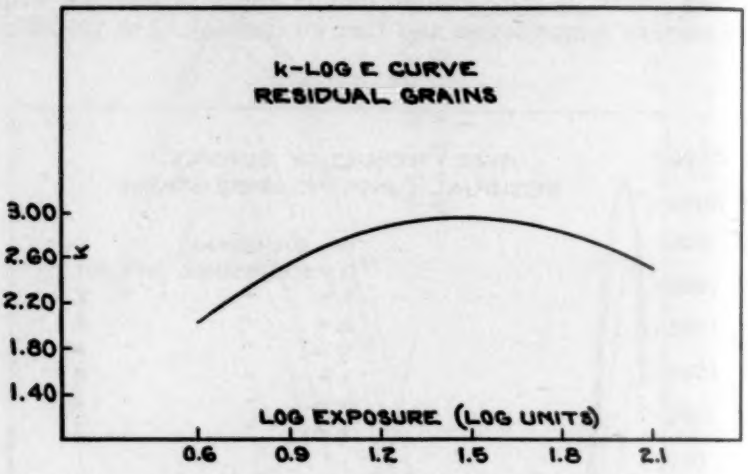


maximum point of the log-log frequency curve, corresponding to the mode of the ordinary frequency curve, is determined by the other two parameters, $\alpha = ln\ x_{Max.}$ and log A. This point is found to shift downward and toward the smaller class sizes with increasing exposure, approximately linearly. The shift of the foci corresponds closely to this. It can then be said that with increasing exposure the log-log size-frequency distribution is both changed in shape and shifted, and in such a way as to preclude the existence of any simple relation between exposure and frequency distribution of residual grains. The impossibility of these data confirming an expo-

sure formula such as previously set up by others is readily seen.

It is contemplated repeating this work using an emulsion better suited for the purpose, which would be one with a greater latitude or range of grain distribution. More com-

FIG. 3.



plete data will be obtained using a smaller increment between exposure steps and also between class sizes. This investigation will also be made for X-ray exposures, in connection with the work of Silberstein and Trivelli already done along this line.[7]

[7] *Phil. Mag.*, 9, 787, 1930.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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OSCILLATIONS OF A DOUBLE SPRING PENDULUM WITH AN ELASTIC-FRICTION COUPLING BETWEEN THE MASSES.

Consider two suspended masses m_1 and m_2 connected by an elastic spring from the point of support to m_1 and an elastic-friction coupling between m_1 and m_2. Let c_1 and c_{12} be the corresponding elastic constants and r the friction constant of the elastic friction coupling. Assume an impressed force $P = A \sin pt$ acting on m_1.

The kinetic energy of the system is

$$T = \tfrac{1}{2}m_1\dot{x}_1^2 + \tfrac{1}{2}m_2\dot{x}_2^2$$

and the potential function of the system is

$$V = \tfrac{1}{2}c_1x_1^2 + \tfrac{1}{2}c_{12}(x_2 - x_1)^2 - m_1gx_1 - m_2gx_2,$$

while the dissipation function is

$$F = \tfrac{1}{2}r(\dot{x}_2 - \dot{x}_1)^2.$$

The equations of motion are, therefore,

$$m_1\ddot{x}_1 = -cx_1 + c_{12}(x_2 - x_1) + r(\dot{x}_2 - \dot{x}_1) + P + m_1g,$$

$$m_2\ddot{x}_2 = c_{12}(x_1 - x_2) + r(\dot{x}_1 - \dot{x}_2) + m_2g.$$

If x_{01} and x_{02} are the equilibrium displacements, then

$$x_1 = x_{01} + \epsilon_1 \qquad \text{and} \qquad x_2 = x_{02} + \epsilon_2$$

and the equilibrium condition is

$$m_1g - c_1x_{01} + c_{12}(x_{02} - x_{01}) = 0, \qquad m_2g - c_{12}(x_{02} - x_{01}) = 0.$$

On substituting these values in the equations of motion we obtain

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 366, vol. 211, March, 1931.

$$m_1\ddot{\xi}_1 = -c_1\xi_1 + c_{12}(\xi_2 - \xi_1) + r(\dot{\xi}_2 - \dot{\xi}_1) + A \sin pt,$$
$$m_2\ddot{\xi}_2 = c_{12}(\xi_1 - \xi_2) + r(\dot{\xi}_1 - \dot{\xi}_2).$$

For the condition of steady oscillations under the impressed periodic force $A \sin pt$ we will assume

$$\xi_1 = \xi_{01} \sin (pt - \psi), \qquad \xi_2 = \xi_{02} \sin (pt - (\psi + \phi)).$$

That is, ξ_{01} lags A by ψ and ξ_{02} and ξ_{01} differ in phase by the angle ϕ.

Considering the corresponding time vector forces, the inertia reactions are

$$-\overline{m_1\ddot{\xi}_1} = \overline{m_1p^2\xi_{01}} \qquad \text{and} \qquad -\overline{m_2\ddot{\xi}_2} = \overline{m_2p^2\xi_{02}},$$

and for the mutual reaction between m_1 and m_2,

$$\overline{c_{12}(\xi_{02} - \xi_{01})} + \overline{rp(\xi_{02} - \xi_{01})} = \text{coupling reaction on } m_1,$$
$$\overline{c_{12}(\xi_{01} - \xi_{02})} + \overline{rp(\xi_{01} - \xi_{02})} = \text{coupling reaction on } m_2.$$

For the equilibrium of the time vectors we have

$$\overline{A} + \overline{m_1p^2\xi_{01}} - \overline{c_1\xi_{01}} + \overline{c_{12}(\xi_{02} - \xi_{01})} + \overline{rp(\xi_{02} - \xi_{01})} = 0,$$
$$\overline{m_2p^2\xi_{02}} + \overline{c_{12}(\xi_{01} - \xi_{02})} + \overline{rp(\xi_{01} - \xi_{02})} = 0,$$

and for the system consisting of masses m_1 and m_2,

$$\overline{A} + \overline{m_1p^2\xi_{01}} + \overline{m_2p^2\xi_{02}} - \overline{c_1\xi_{01}} = 0,$$

where the coupling reaction between m_1 and m_2 is nil due to its mutual effect.

In the projection of these vectors on arbitrarily chosen axis we note the elastic component between m_1 and m_2 is c_{12} times the difference of the projections of the displacement vectors, and the friction or damping component is r times the difference of the projections of the velocity vectors $\overline{p\theta_{01}}$ and $\overline{p\theta_{02}}$ which are at $\pi/2$ phase with the corresponding displacement vectors.

ON THE VIBRATION OF GEARING.

It is well known that in reduction gear drives serious vibrations may occur under certain critical conditions. To simplify the problem at the offset we will consider a single reduction gear drive with a turbine M driving an armature A through a single reduction gear drive. Let I_m = the rota-

tional inertia of the turbine rotor, I_a = the rotational inertia of the armature, and I_p and I_g the moments of inertia of the pinion and gear respectively. Let r_p = the radius of the pinion and r_g the radius of the gear wheel so that the gear ratio is $\gamma = (r_g/r_p)$. Let T_d = the driving torque on the rotor and T_R = the resistance torque on the armature.

If we, at first, neglect the inertias of the pinion and gear, the system has two degrees of freedom. If on the other hand we include the inertias of the pinion and gear the system then has four degrees of freedom. We will designate all coördinates on the pinion shaft by ϕ and all coördinates on the primary or gear shaft by θ. Let ϕ_1 = the actual angular coördinate of the turbine rotor and θ_2 = the angular coördinate of the armature. Also ϕ_p = the angular coördinate of the pinion and θ_g = the angular coördinate of the gear. It is convenient to reduce coördinates on the pinion shaft to hypothetical coördinates on the gear shaft by the ratio of their mean cyclic coördinates of the shafts. That is,

$$\phi_1 = \frac{\partial \phi}{\partial \theta}\theta_1, \qquad \dot{\phi}_1 = \frac{\partial \phi}{\partial \theta}\dot{\theta}_1, \qquad \text{and} \qquad \ddot{\phi}_1 = \frac{\partial \phi}{\partial \theta}\ddot{\theta}_1;$$

$$\phi_p = \frac{\partial \phi}{\partial \theta}\theta_p, \qquad \dot{\phi}_p = \frac{\partial \phi}{\partial \theta}\dot{\theta}_p, \qquad \text{and} \qquad \ddot{\phi}_p = \frac{\partial \phi}{\partial \theta}\ddot{\theta}_p,$$

where

$$\frac{\partial \phi}{\partial \theta} = -\gamma = -\frac{r_g}{r_p}.$$

Let

$T_\phi = c_{1p}(\phi_1 - \phi_p)$ = the elastic torque on the pinion shaft.

$T_\theta = c_{g2}(\theta_g - \theta_2)$ = the elastic torque on the gear shaft.

Case I.—Neglecting the pinion and gear inertias.

The equations of motion are

$$I_m\ddot{\phi}_1 = T_d - T_\phi \qquad \text{and} \qquad I_a\ddot{\theta}_2 = T_\theta - T_R,$$

but since

$$T_\phi \frac{\partial \phi}{\partial \theta} = T_\theta, \qquad T_\phi = -\frac{T_\theta}{\gamma},$$

also

$$\phi_1 = -\gamma\theta_1, \qquad \ddot{\phi}_1 = -\gamma\ddot{\theta}_1.$$

The reduced equations of motion are

$$I_m\gamma^2\ddot{\theta}_1 = -\gamma T_d - T_\theta,$$
$$I_a\ddot{\theta}_2 = T_\theta - T_R,$$

so that on combining, we obtain

$$\left(\frac{I_a \cdot I_m\gamma^2}{I_a + I_m\gamma^2}\right)\frac{d^2}{dt^2}(\theta_1 - \theta_2) = -T_\theta - \frac{\gamma T_d \cdot I_a}{I_a + I_m\gamma^2} + \frac{\gamma^2 T_R I_m}{I_a + I_m\gamma^2}.$$

The coördinate ϕ_1 is related to the coördinate θ_2 by the equation,

$$\phi_1 = -\left[\theta_2 + \frac{T_\theta}{c_{\theta 2}} + \Delta\theta_a\right]\gamma + \frac{T_\phi}{c_{1p}},$$

where $\Delta\theta_g$ is a displacement variation between the teeth due to irregularities of tooth contour, etc. Noting

$$\theta_1 = -\frac{\phi_1}{\gamma} \quad \text{and} \quad T_\phi = -\frac{T_\theta}{\gamma},$$

we have

$$\theta_1 - \theta_2 = T_\theta\left(\frac{1}{c_{\theta 2}} + \frac{1}{\gamma^2 c_{1p}}\right) + \Delta\theta_g;$$
$$\therefore \quad T_\theta = c(\theta_1 - \theta_2) - c\Delta\theta_g.$$

where

$$\frac{1}{c} = \frac{1}{c_{\theta 2}} + \frac{1}{\gamma^2 c_{1p}}.$$

If we let $\theta_1 - \theta_2 = \Delta\theta$, the equation of oscillation is

$$\left(\frac{I_a \cdot I_m\gamma^2}{I_a + I_m\gamma^2}\right)\frac{d^2\Delta\theta}{dt^2} + c\Delta\theta = c\Delta\theta_g - \left(\frac{\gamma T_d \cdot I_a + \gamma^2 T_R \cdot I_m}{I_a + I_m\gamma^2}\right).$$

To account for the irregularity of the tooth contour, we may assume

$$\Delta\theta_g = A \sin n\omega t,$$

where n = no. of teeth and ω = mean angular velocity of gear wheel.

To account for the pulsation in the driving and resistance torque, let

$$T_d = T_{d0} + D \sin m\omega t,$$
$$T_R = T_{R0} + B \sin v\omega t,$$

where

$$T_{d0}\frac{\partial\phi}{\partial\theta} - T_{R0} = -\gamma T_{d0} - T_{R0} = 0.$$

On substituting these values we obtain the equation of the forced oscillation. It is important to note that cyclic irregularities in the gear velocity ratios if agreeing with the natural frequency

$$\tfrac{1}{2}\pi\sqrt{\frac{c(I_a + I_m\gamma^2)}{I_a \cdot I_m\gamma^2}}$$

may cause serious vibrations.

Case II.—Including pinion and gear inertias and the elastic deformation of the gears.

We now have a system with four degrees of freedom. If T is the torque between the gearing pair, then,

Actual.	*Reduced.*
$I_m\ddot{\phi}_1 = T_d - T_\phi$	$I_m\gamma^2\ddot{\theta}_1 = -\gamma T_d - T_{\phi\theta}$,
$I_p\ddot{\phi}_p = T_\phi + \frac{T}{\gamma}$,	$I_p\gamma^2\ddot{\theta}_p = T_{\phi\theta} - T$,
$I_g\ddot{\theta}_g = T - T_\theta$,	$I_g\ddot{\theta}_g = T - T_\theta$,
$I_a\ddot{\theta}_2 = T_\theta - T_R$,	$I_a\ddot{\theta}_2 = T_\theta - T_R$,

where

$$T_{\phi\theta} = T_\phi\frac{\partial\phi}{\partial\theta} = -\gamma T_\phi, \qquad T = Pr_g, \qquad \frac{T}{\gamma} = Pr_p,$$

P being the tangential tooth pressure.

For the displacement between the gear pair, let $1/c_{gg}$ = the actual angular displacement per unit torque for the gear wheel due to its local elasticity, and $1/c_{pp}$ = the actual angular displacement per unit torque to account for the elasticity of the pinion alone. Let $\Delta\theta_{gg}$ and $\Delta\theta_{pp}$ be the corresponding plays. Then,

$$\phi_p = -\gamma\left(\theta_g + \frac{T}{c_{gg}}\right) - \gamma\Delta\theta_{gg} - \frac{T}{\gamma c_{pp}} + \Delta\theta_{pp};$$

$$\therefore\quad \theta_p = -\frac{\phi_p}{\gamma} = \theta_g + \frac{T}{c_{gg}} + \Delta\theta_{gg} + \frac{T}{\gamma^2 c_{pp}} + \Delta\theta_{pp},$$

$$\theta_p - \theta_g = T\left(\frac{1}{c_{gg}} + \frac{1}{\gamma^2 c_{pp}}\right) + \Delta\theta_{gg} + \Delta\theta_{pp}.$$

Thus the reduced elastic constant and plays gre

$$\frac{\mathbf{I}}{c_{gp}} = \frac{\mathbf{I}}{c_{gg}} + \frac{\mathbf{I}}{\gamma^2 c_{pp}}, \qquad \text{and} \qquad \Delta\theta_{gp} = \Delta\theta_{gg} + \Delta\theta_{pp};$$

$$\therefore \quad \theta_p - \theta_g = \frac{T}{c_{gp}} + \Delta\theta_{gp}$$

and

$$T = c_{gp}(\theta_p - \theta_g) - c_{gp}\Delta\theta_{gp}.$$

We also note

$$T_{\phi\theta} = c_{1p}(\phi_1 - \phi_p)\frac{\partial\phi}{\partial\theta} = -\gamma c_{1p}(\phi_1 - \phi_p)$$

$$= \gamma^2 c_{1p}(\theta_1 - \theta_p),$$

since

$$\phi_1 = -\gamma\theta_1, \qquad \text{and} \qquad \phi_2 = -\gamma\theta_2.$$

The reduced equations of oscillation are

$$I_m\gamma^2\ddot{\theta}_1 = -\gamma T_d + \gamma^2 c_{1p}(\theta_p - \theta_1), \tag{1}$$

$$I_p\gamma^2\ddot{\theta}_p = \gamma^2 c_{1p}(\theta_1 - \theta_p) + c_{gp}(\theta_g - \theta_p) + c_{gp}\Delta\theta_{gp}, \tag{2}$$

$$I_g\ddot{\theta}_g = c_{gp}(\theta_p - \theta_g) + c_{g2}(\theta_2 - \theta_g) - c_{gp}\Delta\theta_{gp}, \tag{3}$$

$$I_a\ddot{\theta}_2 = c_{g2}(\theta_g - \theta_2) - T_R, \tag{4}$$

where, as before,

$$\Delta\theta_{gp} = A \sin n\omega t,$$

for tooth irregularities

$$T_d = T_{d0} + D \sin m\omega t, \qquad T_R = T_{R0} + B \sin v\omega t$$

with the equilibrium value,

$$T_{d0}\frac{\partial\phi}{\partial\theta} - T_{R0} = -\gamma T_{d0} - T_{R0} = 0.$$

We will now consider the derivation of the equations by the Lagrangian method. The kinetic energy is

$$K = \tfrac{1}{2}I_m\dot{\phi}_1^2 + \tfrac{1}{2}I_p\dot{\phi}_p^2 + \tfrac{1}{2}I_g\dot{\theta}_g^2 + \tfrac{1}{2}I_a\dot{\theta}_2^2$$

and the potential energy is

$$V = \tfrac{1}{2}c_{1p}(\phi_1 - \phi_p)^2 + \tfrac{1}{2}c_{gp}\overline{\Delta\theta_{ge}^2} + \tfrac{1}{2}c_{g2}(\theta_g - \theta_2)^2,$$

where $\Delta\theta_{ge}$ is the elastic deflection between the gear pair and reduced to the gear shaft θ, and c_{gp} is obtained from the relation,

$$\frac{1}{c_{gp}} = \frac{1}{c_{gg}} + \frac{1}{\gamma^2 c_{pp}}.$$

Now

$$\begin{aligned}\phi_p &= \frac{\partial\phi}{\partial\theta}\theta_g + \frac{\partial\phi}{\partial\theta}(\Delta\theta_{ge} + \Delta\theta_{gp}) \\ &= -\gamma\theta_g - \gamma(\Delta\theta_{ge} + \Delta\theta_{gp}); \\ \therefore\quad \Delta\theta_{ge} &= -\frac{\phi_p}{\gamma} - \theta_g - \Delta\theta_{gp},\end{aligned}$$

so that

$$V = \tfrac{1}{2}c_{1p}(\phi_1 - \phi_p)^2 + \tfrac{1}{2}c_{gp}\left(\frac{\phi_p}{\gamma} + \theta_g + \Delta\theta_{gp}\right)^2 + \tfrac{1}{2}c_{g2}(\theta_g - \theta_2)^2.$$

The Lagrangian equations of motion, noting T_d and T_R, are the applied forces on the coördinates ϕ_1 and θ_2 respectively, are

$$\begin{aligned}I_m\ddot{\phi}_1 &= c_{1p}(\phi_p - \phi_1) + T_d, \\ I_p\ddot{\phi}_p &= c_{1p}(\phi_1 - \phi_p) - \frac{c_{gp}}{\gamma}\left(\frac{\phi_p}{\gamma} + \theta_g + \Delta\theta_{gp}\right), \\ I_g\ddot{\theta}_g &= -c_{gp}\left(\frac{\phi_p}{\gamma} + \theta_g + \Delta\theta_{gp}\right) + c_{g2}(\theta_2 - \theta_g), \\ I_a\ddot{\theta}_2 &= c_{g2}(\theta_g - \theta_2) - T_R.\end{aligned}$$

On substituting $\ddot{\phi}_1 = -\gamma\ddot{\theta}_1$, $\ddot{\phi}_p = -\gamma\ddot{\theta}_p$ we obtain

$$\begin{aligned}I_m\gamma^2\ddot{\theta}_1 &= c_{1p}\gamma^2(\theta_p - \theta_1) - \gamma T_d, \\ I_p\gamma^2\ddot{\theta}_p &= c_{1p}\gamma^2(\theta_1 - \theta_p) + c_{gp}(\theta_g - \theta_p + \Delta\theta_{gp}), \\ I_g\ddot{\theta}_g &= c_{gp}(\theta_p - \theta_g - \Delta\theta_{gp}) + c_{g2}(\theta_2 - \theta_g), \\ I_a\ddot{\theta}_2 &= c_{g2}(\theta_g - \theta_2) - T_R,\end{aligned}$$

which agrees with the previous derivation.

Generalizations.—We have noted certain simplifications are effected by a reduction through mean cyclic coördinates, i.e., proportional to the gear ratios to the primary shaft. Thus, if θ corresponds to the primary shaft we have

$$\phi_n = \frac{\partial\phi}{\partial\theta}\theta_n, \qquad \dot{\phi}_n = \frac{\partial\phi}{\partial\theta}\dot{\theta}_n, \qquad \ddot{\phi}_n = \frac{\partial\phi}{\partial\theta}\ddot{\theta}_n,$$

$$T_{\phi n}\frac{\partial\phi}{\partial\theta} = T_{\theta n}, \qquad I_{\phi n}\ddot{\phi}_n\frac{\partial\phi}{\partial\theta} = I_{\phi n}\left(\frac{\partial\phi}{\partial\theta}\right)^2\ddot{\theta}_n = I_{\theta n}\ddot{\theta}_n,$$

$$c_{\phi mn}(\phi_m - \phi_n)\left(\frac{\partial\phi}{\partial\theta}\right) = c_{\phi mn}\left(\frac{\partial\phi}{\partial\theta}\right)^2(\theta_m - \theta_n) = c_{\theta mn}(\theta_m - \theta_n);$$

$$\therefore \quad c_{\theta mn} = c_{\phi mn}\left(\frac{\partial\phi}{\partial\theta}\right)^2, \qquad \text{and} \qquad I_{\theta n} = I_{\phi n}\left(\frac{\partial\phi}{\partial\theta}\right)^2,$$

i.e.

$$\left|\begin{matrix} c_{\theta mn} = c_{\phi mn}\gamma_{\phi\theta}^2 \\ I_{\theta n} = I_{\phi n}\gamma_{\phi\theta}^2 \end{matrix}\right|,$$

where

$$\frac{\partial\phi}{\partial\theta} = \frac{\partial\phi}{\partial\psi}\cdot\frac{\partial\psi}{\partial\theta} = (-\gamma_{\phi\psi})(-\gamma_{\psi\theta}) = \gamma_{\phi\psi}\gamma_{\psi\theta}, \text{ etc.}$$

The kinetic energy of system of gearing masses is

$$T = \Sigma\tfrac{1}{2}I_n\left(\frac{\partial\phi}{\partial\theta}\right)^2\dot{\theta}_n^2 = \Sigma\tfrac{1}{2}I_n\gamma_{\phi\theta}^2\dot{\theta}_n^2$$

and the potential function is

$$V = \Sigma\tfrac{1}{2}c_{\phi mn}\left(\frac{\partial\phi}{\partial\theta}\right)^2(\theta_m - \theta_n)^2 = \Sigma\tfrac{1}{2}c_{\phi mn}\gamma_{\phi\theta}^2(\theta_m - \theta_n)^2,$$

where $\dfrac{\partial\phi}{\partial\theta} = -\gamma_{\phi\theta}$ = the velocity ratio of the mean cyclic velocities of the shafts with respect to the primary shaft.

OSCILLATION OF A SPRING CONNECTED MASS CONSTRAINED BY A ROTATING FRAME.

Let a = the radial distance to spring seat,
l = unstrained length of spring,
$x + x_0$ = total deflection of spring,
x_0 = kinetic equilibrium extension of spring,
c = spring constant,
ω = angular velocity of rotating frame,
m = spring connected mass and constrained by radial guides,
$r = a + 1 + x_0 + x$ = radial distance to mass.

Introducing the centrifugal loading, we have

$$m\left(\frac{d^2r}{dt^2} - \omega^2 r\right) = -c(x + x_0),$$

where

$$r = a + l + x_0 + x.$$

The kinetic equilibrium position is

$$m\omega^2(l + a + x_0) = cx_0,$$

so that the equation of motion is

$$m\left(\frac{d^2x}{dt^2} - \omega^2 x\right) = -cx,$$

or

$$m\frac{d^2x}{dt^2} + (c - m\omega^2)x = 0,$$

with the frequency of oscillation given by

$$f = \frac{1}{2\pi}\sqrt{\frac{c - m\omega^2}{m}}.$$

Integrating, we obtain

$$\frac{m\dot{x}^2}{2} + \frac{(c - m\omega^2)x^2}{2} = c.$$

By the Lagrangian method,

$K = \frac{1}{2}m(\dot{r}^2 + \omega^2 r^2)$ for the K.E.,

$V = \frac{1}{2}c(r - (l + a))^2$ for the Potential Energy.

Then

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{r}}\right) = m\ddot{r}, \qquad \frac{\partial K}{\partial r} = m\omega^2 r,$$

$$\frac{\partial V}{\partial r} = c(r - (l + a));$$

$$\therefore \quad m(\ddot{r} - \omega^2 r) = -c(r - (l + a)),$$

where

$$m\omega^2(a + l + x_0) = cx_0, \qquad r = a + l + x_0 + x;$$

$$\therefore \quad m(\ddot{x} - \omega^2 x) = -cx,$$

as before.

In place of the total kinetic energy we may introduce the *relative* kinetic energy $K_r = \frac{1}{2}m\dot{r}^2$ provided we include a *centrifugal function*,

$$\Phi_r = -\frac{1}{2}m\omega^2 r^2.$$

Then,

$$\frac{d}{dt}\left(\frac{\partial K_r}{\partial \dot{r}}\right) = -\frac{\partial V}{\partial r} - \frac{\partial \Phi_r}{\partial r}.$$

In the application of the energy principle we must include the reaction of the *moving* constraint, N. We note, from the principle of angular momentum,

$$Nr = \frac{d}{dt}(m\omega r^2) = 2m\omega r\frac{dr}{dt};$$

$$\therefore \quad N = 2m\omega\dot{r},$$

which is the reaction of the Coriolis component due to motion in a rotating frame. Then

$$Nr\omega dt - dV = dK,$$

$$(2m\omega\dot{r})r\omega dt - c(r - (l + a))dr = m\dot{r}d\dot{r} + m\omega^2 r dr,$$

from which we obtain on dividing by dt,

$$m(\ddot{r} - \omega^2 r) = -c(r - (l + a)).$$

For a more complicated case we will consider the shaft torque to be of the form

$$T = A + B \sin pt.$$

The rotation of the frame will then be defined by the coordinate θ. The system has two degrees of freedom, r and θ. Then

$$K = \tfrac{1}{2}[m(\dot{r}^2 + r^2\dot{\theta}^2) + I_f\dot{\theta}^2], \qquad V = \tfrac{1}{2}c(r - (l + a))^2,$$

where I_f is the moment of inertia of the frame and the applied force acting on the coördinate θ is T.

The equations of motion are

$$m(\ddot{r} - r\dot{\theta}^2) = -c(r - (l + a)),$$

$$(I_f + mr^2)\ddot{\theta} + 2m\dot{r}\dot{\theta}\cdot r = A + B \sin pt.$$

To study the oscillations, let

$$r = r_0 + \xi, \qquad \dot{\theta} = \omega + \eta;$$

$$\therefore \quad \dot{r} = \dot{\xi}, \qquad \dot{\theta}^2 = \omega^2 + 2\omega\eta, \qquad r\dot{\theta}^2 = r_0\omega^2 + 2r_0\omega\eta + \xi\omega^2.$$

The equations of oscillation are

$$m\ddot{\xi} + (c - m\omega^2)\xi - 2mr_0\omega\eta = 0, \tag{1}$$

$$(I_f + mr_0^2)\dot{\eta} + 2m\omega r_0\xi = A + B \sin pt. \tag{2}$$

where,

$$mr_0^2\omega^2 = c(r_0 - (l + a)).$$

Differentiating (2) and for a solution if we let

$$\xi = M \cos pt \qquad \text{and} \qquad \eta = N \cos pt,$$

we obtain

$$-mMp^2 + (c - m\omega^2)M - 2mr_0\omega N = 0,$$

$$-(I_f + mr_0^2)Np - 2m\omega r_0 M = B.$$

(To be continued)

Earth-Resistivity Survey at Huancayo, Peru. W. J. ROONEY and O. H. GISH. (*Terr. Mag. and Atmospheric Elec.*, June, 1930.) For a long time it has been known that electric currents flow in the crust of the earth. They were studied only by making observations of the potential gradient in the earth, and for a long time no detailed measurements were made of the resistance of the material through which they flowed. "While the strength of such currents does depend, as was supposed, on the average resistivity of large portions of the Earth's crust, the significance of potential measurements made at any point will depend largely on local variations of resistivity in the region of the determinations, since, for a given current, the recorded potential will be directly proportional to the resistance of the ground in that region." The Department of Terrestrial Magnetism of the Carnegie Institution therefore developed apparatus for the measurement of earth resistance and has made surveys of its value near the three places where it maintains a program of continuous registration of earth-current potentials, the Ebro Observatory, Spain, Watheroo, Western Australia, and Huancayo, Peru. This paper summarizes the Huancayo results.

The region surveyed, about 25 km.^{2} in area, is about 160 km. east of Lima, Peru, and lies about 3300 m. above the sea, in a valley between two mountain ranges. On each of two open flat terraces, Pampa Paccha and Pampa Sicaya, were stretched the north-south and the east-west lines for studying the earth currents. In addition about 600 measurements of earth resistance were made on the two pampas, part in the dry season and part when the wet season was under way. The results about to be quoted are the "Average resistivity to depth approximately equal to electrode-separation." On Pampa Sicaya "near the surface the resistivity ranged from 112,000 to about 1500 ohm-cm. The highest values are all from the southern part of the pampa where the coarse gravel was found and the lowest ones from the northern part where the surface soil was clay. The mean values fall off sharply with depth from 60 to 200 meters and more gradually as the depth is still further increased." 10,000 ohm-cm. is the value toward which the results tend to converge. The other pampa furnished results not greatly different. On neither pampa was any subsurface water found by the digging of wells. In opposition to what would be expected we read in relation to the observations made in the flood-meadows of one stream and along the course of another "the recorded resistivities and their variations were similar to those found on the pampas." A comparison of values obtained in the dry and in the wet season showed that "on the whole the results indicate that the seasonal variation is small and relatively unimportant when any appreciable depth is considered. Aside from the shallow-depth measurements the differences recorded were only a few per cent." G. F. S.

COMMITTEE ON SCIENCE AND THE ARTS.

ABSTRACTS OF REPORTS.

SELF-ALIGNING COUPLING.

HALL OF THE INSTITUTE,
PHILADELPHIA, March 6, 1929.

No. 2901.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the Self-Aligning Coupling invented by Mr. Gustave Fast, of Annapolis, Maryland, reports as follows:

The Fast flexible, or self-aligning, coupling, which is the subject of this report, is a device for coupling together the ends of two shafts for the transmission of power when the shafts are not in exact alignment.

This coupling differs from other flexible couplings in that no flexible material is used in its construction, its flexibility depending on the way its parts are arranged.

The fundamental patent, No. 1,356,860, granted on October 26, 1920 to Gustave Fast, of Mount Washington, Maryland, contains a drawing from which Plate I was made. In Fig. 1 of this Plate, *1* represents the two members that are keyed to the ends of the shafts to be connected, each of these members having teeth at *2* meshing with teeth on the interior of the annular casing *3* that surrounds these members.

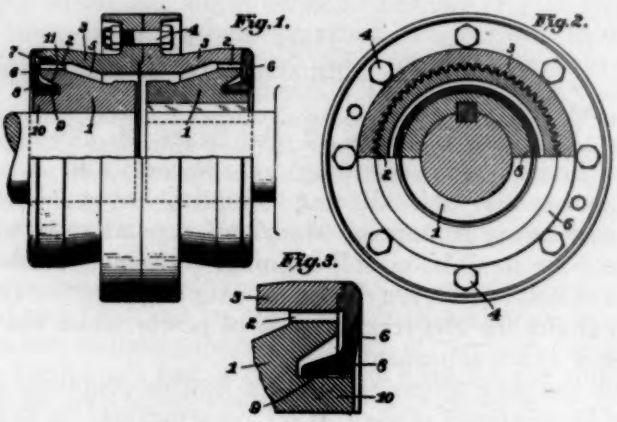
This casing is in two parts and when installed these are bolted together as at *4*.

Clearance is provided between the teeth of members *1* and *3* and the space *5* between them is filled with a heavy oil.

At each end of casing *3* a ring *6* is fastened, these rings having a flange *8* extending inwardly. These flanges are seated at *9* on cylindrical extensions of member *1*. This is shown in detail in Fig. *3* in which it is seen that the bearing surface of *9* is rounded longitudinally, recess *11* being provided to give play to the combination.

Any error in the alignment of the shafting is divided between the two rings *6* which may tip slightly with relation to members *1* on account of the rounded face *9*. When this takes place the clearance at the teeth *2* compensates for the changes in position. Since space *5* slopes outward, centrifugal force will throw the lubricating oil out against the teeth and into the bearings at *9*.

PLATE I



Twenty claims are allowed in this patent of which the first is:

"A flexible coupling for shafts comprising members fixed to the shaft ends, a member encircling the shaft members and having driving means engaging therewith to impart rotary movement from one shaft to the other, said encircling member being capable of movement in a radial plane relative to the shaft members, and bearings independent of the driving means between the encircling member and the shaft member maintaining contact in all relative positions of the encircling member and shaft members, substantially as described."

A reissue of this patent was granted on August 16, 1921. This contains an additional description of the location of the bearing *9* and nine additional claims are granted.

Four other patents have been granted to Mr. Fast on this coupling. These have to do with its details, one covering a form of construction that permits adjustment of one shaft in the direction of its length, and another provides a con-

struction by means of which the coupling may be set to connect or disconnect the shafts.

The inventor submitted a sample of his coupling to the Institute's Committee with a section cut out to show the relation of the ends of the shafts to the parts of the coupling. An inspection of it shows how it corrects for either angular or offset misalignment or for a combination of both.

The design of this coupling permits it to be used for the transmission of heavy loads and the manufacturer publishes a list of couplings especially adapted for heavy duty. These extend to a capacity of 12,000 H.P. per 100 R.P.M. with a maximum of 500 R.P.M. This is for a coupling $47\frac{1}{2}$ inches long and having an oil capacity of $7\frac{1}{2}$ gallons.

The centrifugal method of distributing the oil in this coupling insures a continuous film between the bearing surfaces that reduces wear on the load carrying surfaces and provides a practically noiseless transmission of power.

The error between the gear faces of Fast's Coupling is only a small fraction of the error of alignment between the shafts. This may be seen by a study of Plate II in which

PLATE II

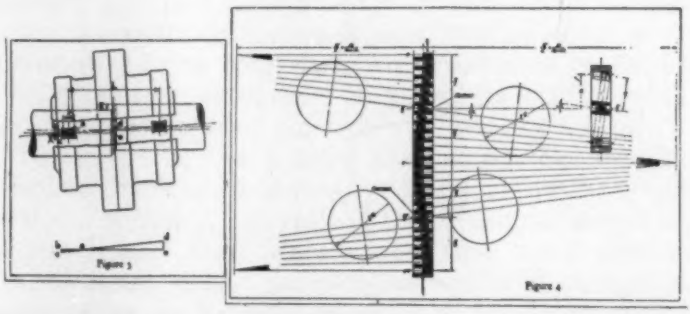


Fig. 3 shows two shafts that are parallel but out of line by an amount E. The distance between the center of the gear faces is L and the width of the gear faces is f.

From the similar triangles abc and ade we get $bc : de = ab : ae$, hence $bc = \dfrac{de \times ab}{ae}$. The resulting misalignment in the gear face is equal to $2bc$ and may be called X. $de = \frac{1}{2}E$.

$ab = \frac{1}{2}f$, $ae = \frac{1}{2}L$ and $bc = \frac{1}{2}X$. By substituting these values in the expression for bc and eliminating the factor $\frac{1}{2}$ we get $X = \frac{E \times f}{L}$ or $\frac{f}{L}E$. Since the ratio $\frac{f}{L}$ is usually made 1/10, $X = 1/10E$. This means that the error of alignment between the gear faces is only 1/10 that of the shaft alignment. This actually makes only a slight opening of the gear teeth surfaces which is largely taken care of by the lubrication.

Figure 4 of this Plate shows the development of an exaggerated misalignment of the gear teeth in which the function of the lubricant is seen.

The revolving of the coupling produces centrifugal force and this causes the oil to be spread over the inner circumference of the casing and forces it between the teeth of the gears. This figure shows two main points of maximum pressure at $\pi/2$ and at $3\pi/2$, 180° apart.

In consideration of the novelty of design of this coupling, the stability of its construction and its successful practical application, THE FRANKLIN INSTITUTE awards its JOHN PRICE WETHERILL MEDAL to MR. GUSTAVE FAST, of Annapolis, Maryland.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

ACCURATE RADIO FREQUENCY TRANSMISSIONS ON 5,000 KC.

In an announcement by the bureau published at the beginning of the year appearing in several technical magazines and the newspapers (Technical News Bulletin No. 165, p. 8; January, 1931) mention was made of the inauguration of an improved type of standard frequency transmission upon 5,000 kc. These signals are transmitted from 1:30 to 3:30 and 8:00 to 10:00 P.M., eastern standard time, three Tuesdays in the month. They represent a step in the program of ultimately providing continuous transmissions of accurately known frequency available in all parts of the United States.

The transmitting set is in a temporary location at College Park, Maryland, ten miles from the Bureau of Standards. The transmitted frequency is controlled by a standard piezo oscillator at the transmitter. Work has been begun on facilities for control of the transmissions by the primary frequency standard located at the bureau's main laboratory. As the transmitting station is now located adjacent to an experimental flying field, there are restrictions on the height at which an antenna may be built, and its location, with the result that the radiated power is now severely limited.

The power in the present antenna system, which is a horizontal dipole one-eighth wave-length above the ground, is less than 200 watts. In spite of the limitations stated above, the signals have been utilized more than 2,000 miles away. This statement should not be construed as meaning that the transmissions are heard everywhere within a radius of 2,000 miles, because some reports from points a few hundred miles distant indicate that the reception there may not be reliable at the present times of transmission. Radio reception varies with the distance, time of day, season of the year, and frequency. It is likewise known to vary from year to year. A consideration of all of these elements affecting the trans-

* Communicated by the Director.

mission shows the magnitude of the problem of supplying universally available standard frequency service.

In working out the problem the bureau has asked for assistance from the many people engaged in radio work in various Government bureaus, the Army, the Navy, radio companies and measurement laboratories, and others. The bureau desires reports on the reception of these signals in various localities. The information desired includes time of reception, approximate field intensity, and degree of fading. The bureau appreciates the coöperation evidenced by the many reports which are being received, and which are helpful in the plans to improve the standard frequency service.

The accuracy of the 5,000-kc. transmissions is within one part in a million. Plans for the expansion of the service cover the following, which are being undertaken as rapidly as possible: increase in power output, transmission on one or two additional higher frequencies, further increase in accuracy, and extension of time of transmissions.

THE ARC SPECTRUM OF ZIRCONIUM.

Nearly 1,600 lines have been measured at the bureau in the arc spectrum of zirconium between 2,085 A. in the ultra-violet and 9,300 A. in the infra-red. Approximately 80 per cent. of these lines, which includes all but 7 for which temperature classes have been determined, have been classified as combinations between terms of the singlet, triplet, and quintet systems. The terms, without exception, are those required by Hund's theory, and many of them have been confirmed by Zeeman effect observations. Many pairs of terms of nearly the same value, with the same inner quantum number, and arising from the same electron configuration, mutually perturb one another with the result that lines originating in them exhibit abnormal intensities and Zeeman effects. The lowest term of the spectrum, a^3F, and the lowest term of the quintet system, a^5F, both form Rydberg series with higher terms, from which the distance separating the ground states of ZrI and ZrII proves to be 56,077 cm.$^{-1}$, giving an ionization potential of 6.92 volts for the neutral Zr atom. The resonance lines are given by the combination $a^3F - z^5G^\circ$, whereas the

raies ultimes are given by $a^3F - z^3G^\circ$. More than 200 Zr lines have been identified with dark lines in the solar spectrum, of which nearly half represent term combinations which do not appear in emission spectra observed in the laboratory.

These measurements will be reported in detail in the April number of the *Bureau of Standards Journal of Research.*

THE VIBRATION OF U-BARS.

A theoretical study has been made at the bureau of elongated and short U-bars with special reference to their use as vibrators in investigations of elastic hysteresis.

First, an expression for the frequency of the fundamental mode of vibration of the elongated U-bar is derived by solving the differential equations of motion of the yoke and of the prongs through the use of an approximation. The physical basis for the approximation is the fact that the bar vibrates with a pitch differing slightly from that of the clamped-free bar of half the length.

Secondly, Ritz's method of approximation is developed for initially curved bars, the development being based on the principle of least action. In this connection a new proof is given of Rayleigh's method for determining the fundamental mode of vibration. As an illustration of Ritz's method, the example of the free-free bar is treated, and the results of the calculation are compared with the known solution of this problem. Next, the method is used to determine the fundamental mode of vibration of the short U-bar, such a bar being defined as one wherein the length of the curved portion, or the yoke, is equal to the sum of the lengths of the two parallel prongs.

Finally, Rayleigh's method is used to determine the static deformation of the short U-bar, produced by the application of a single load so applied that the deformation is most nearly that present in the fundamental mode of vibration.

This investigation will be more fully discussed in the April number of the *Bureau of Standards Journal of Research.*

HEAT TREATMENT OF GLASS.

A paper to be published in the April number of the *Bureau of Standards Journal of Research* shows some of the variations found in the heating curves of a glass when it is chilled or annealed in varying degrees. A discussion is included which is designed to explain the relation of these variations to certain assumed changes in the physico-chemical condition of glass as its "effective" annealing temperature is varied. The relation which these changes in condition bear to the properties of a glass and to some phases of the problem of annealing is also considered. It is evident from both the experimental results and these discussions that annealing procedures may be so manipulated that certain properties of the glass may often be made either more desirable or more nearly coincident with standards of requirement than they would be if a fixed (or indefinite) annealing schedule were always followed for a given type of glass.

GASES EVOLVED BY FELDSPAR DURING HEATING.

The Columbus Branch of the bureau is now studying the gases evolved by some feldspars during heating. The feldspar samples used are the same as those which the bureau has been studying extensively and for which data were published in the *Journal of the American Ceramic Society*, January, 1931. It has been known for a considerable time that some feldspars fuse to a clear bubble-free glass while others form only an opaque glassy mass, the opacity presumably being the result, in some cases, of inclosed bubbles containing gas. The investigation has for its purpose the determination of the amount of gases given off as well as some indication of their relative compositions.

The procedure and apparatus used are similar to those developed at the Washington laboratory for the determination of the gaseous constituents of metals. A ten gram sample is heated in a fused silica test tube, contained within a larger fused silica tube which is closed at one end. A platinum resistance furnace is used. The open end of the large fused silica tube is sealed by means of a brass unit to a glass vacuum apparatus. The apparatus is evacuated to about 2 mm.

pressure before the furnace is heated. The gases evolved during the heating are continuously circulated by means of a mercury vapor pump through a closed circuit containing (1) a phosphorus pentoxide absorption tube which removes the water and (2) an absorption tube containing ascarite (sodium-hydroxide-asbestos absorbent) which removes the acid anhydrides, carbon dioxide, sulphur dioxide, etc. Means are provided for cutting out either absorption tube from the closed circuit. The gain in weight of the tubes gives the quantitative determination of the gases.

Nineteen feldspars have been used in determinations up to 1,000° C. and several samples have been rechecked. The total weight of gases drawn off at 1,000° C. from the various feldspar samples varies from 0.2 per cent. up to 1.0 per cent. and follows closely the loss on ignition of the feldspar. It has been found that the gases may contain from 35 to 96 per cent. (weight percentage) water vapor but usually between 70 and 85 per cent. The acid anhydrides are nearly always present in determinable amounts and vary from 0.1 up to 55 per cent. and usually between 4 and 12 per cent. The other gases comprise the remainder and vary from 3.5 to 19 per cent. but usually lie between 7 and 15 per cent. This indicates that the loss on ignition above 110° C. is in most cases not all water vapor and that in calculating chemical analyses back to mineralogical composition we are not justified in using all the loss on ignition as a basis for calculation of kaolinite. This explains why in calculated mineralogical compositions of feldspar more kaolinite is obtained than is actually found to be present by microscopic methods.

EFFECT OF PROLONGED BOILING, AUTOCLAVE TREATMENT, AND DRYING ON THE WATER ABSORPTION OF EARTHENWARE.

This phase of the work was undertaken to determine:

(1) Whether or not representative earthenware could be completely saturated with water by the commonly used boiling method.

(2) The rate at which absorption progressed.

(3) What percentage of the pores could be filled in the event that absorption did reach or approach completion.

Samples broken from two plates of each of sixteen brands were used and were boiled progressively for 240 hours. Weighings were taken at 1, 2, 5, 10, and 20 hour intervals and thereafter at 20 hour intervals. The specimens were then immersed in water and autoclaved at 150 lbs./in.^{2} for eleven hours.

After the 240 hours boiling and 11 hours autoclave treatment the average absorption for the 16 brands was 9.97 per cent. The absorption after 200 hours of boiling was 9.64 per cent., after 120 hours 9.15 per cent., after 80 hours 8.72 per cent., after 40 hours 8.22 per cent., after 10 hours 7.6 per cent., and after 5 hours 7.4 per cent. Therefore, the present standard 5 hour boiling test is approximately 75 per cent. as efficient as 240 hours boiling plus 11 hours autoclave treatment. That the gain in weight was caused primarily by mechanical filling of the pores, and not by absorption or possibly rehydrating, is evidenced by the fact that nearly all of the water taken up by the specimens could be driven off at 110° C. Average values for the 16 brands showed only 1.34 per cent. moisture remaining after 1 hour or drying at 110° C. and 0.31 per cent. after 6 hours drying. The percentage of total pores not filled by this 251 hour treatment was calculated by the following formula for duplicate specimens of four brands of ware:

$$p = 1 - \left\{\frac{W_w - W_d}{V_b - V_c}\right\} \cdot 100,$$

where

p = per cent. unfilled pores,
W_w = weight of saturated specimen,
W_d = weight of dry specimen,
V_b = bulk volume,
V_c = calculated solid volume $= \dfrac{W_d}{\text{true specific gravity}}$.

The values obtained were: Brand 2—19.5 and 19.3 per cent.; Brand 8—20.8 and 21.0 per cent.; Brand 14—11.5 and 11.3 per cent.; Brand 15—20.0 and 21.8 per cent.

In order to determine the efficiency of autoclave treatment as compared with boiling, two specimens of six different brands were dried at 110° C. and subjected for ten hours to

the action of water under 150 lbs./in.^{2} pressure, the specimens being immersed. The following results were obtained:

Water Absorption after	Brand Numbers.					
	4	5	8	9	10	15
240 hours boiling (per cent.)	12.0	11.7	10.0	10.1	10.9	10.0
10 hours autoclave treatment (per cent.)	11.0	11.8	10.2	9.3	11.0	11.2
Difference	−1.0	+0.1	+0.2	−0.8	+0.1	+1.2

These values indicate that a 10 hour autoclave treatment at 150 lbs./in.^{2} pressure may be as effective as 240 hours of boiling in filling the pores of typical earthenware bodies.

CUTLERY MARKING OF CHINAWARE.

A preliminary item on this investigation was published in Technical News Bulletin No. 153 (January, 1930). As explained in this item, the attention of the bureau has been directed a number of times to the so-called cutlery marking of tableware. These marks resemble lines made with a lead pencil on paper and can be made without exerting any more pressure on the metal, such as a silver-plated knife, than would normally be used in drawing the knife across a plate.

The preliminary investigations showed that an excess of carbon in a furnace atmosphere at 860° C. would not produce a surface susceptible to metal marking even though the glaze had been darkened sufficiently, by reduction of the lead, to make the ware unmarketable. Preliminary work with undetermined concentrations of introduced SO_2 showed that surfaces susceptible to marking could be produced by "sulphuring." Further experiments with measured concentrations of introduced SO_2 at 860° C. for one-half hour gave the following results:

1. Concentrations of SO_2 gas in the furnace atmosphere as high as 1.2 per cent. did not produce a surface on any of the specimens tested which could be metal-marked.

2. The only glaze showing harmful defects (blisters) after exposure to from 0.1 to 1.2 per cent. SO_2 was on a colored vitrified body.

3. Increasing the concentration of SO_2 from 0.1 to 1.2 per cent. increased the susceptibility of the surfaces to scratching with metal, particularly with nickel and steel.

4. Three per cent. SO_2 in the furnace atmosphere was sufficient to dull the glaze on two typical hotel ware and two earthenware bodies.

5. Concentrations of SO_2 of 3 and 5 per cent. caused the glaze on one brand of vitrified, low-fired ware to metal-mark, but produced no blistering or scumming. Concentrations of 7.5, 10, and 12.5 per cent. produced a slight scum, impaired the gloss, the metal marks could not be entirely removed by washing from the specimens exposed to a concentration of 12.5 per cent., and the scum formed was composed at least partly of lead sulphate.

In the course of this study a secondary phenomenon was observed which would be of considerable importance to one studying cutlery marking. It was noted that the surfaces could be marked after every test if the marking was tried shortly after the specimens were removed from the oven and before they had been washed. Sufficient evidence was obtained to indicate that the glass surfaces dried by heating at temperatures above 400° C. will mark, whereas a surface protected with a film of adsorbed moisture, acting as a lubricating film, will not mark.

ERIOMETER FOR GRADING WOOL.

In the grading of wool for manufacturing purposes the average diameter of the fibers is a dominant dimensional characteristic of the material which serves as the present basis for the classification. Sets of wool standards have been established for the industry based entirely on the average diameter of the fibers. In present trade practice the grading is done by men of long experience in the industry who, by merely observing and handling the material, assign it intuitively to its proper grade. There is no suitable instrument available for the rapid, convenient, and accurate evaluation of wool on the established basis.

A simple rapid optical method for measuring the average diameter of a group of fibers was devised by Thomas Young in

1824, but no thorough investigation has ever been made of the practical possibilities of this method in the grading of wool. The method is based on the phenomenon of diffraction, which is manifested when a line-source of light is viewed through a bundle of fibers which are arranged approximately parallel to one another and to the source. Under these conditions a banded light distribution is seen extending laterally and symmetrically on both sides of the source, and the separation of the bands is—to a close approximation—inversely proportional to the average diameter of the fibers.

In a paper to be published in the *Bureau of Standards Journal of Research* for April a new construction of Young's instrument (the eriometer) is described in which a stationary artificial light source is used and more convenient and accurate means supplied for determining the separation of the diffraction bands. Comparative diameter measurements are made with the eriometer and with a microscope on wool and other textile fibers. The samples studied cover a range of average diameters from 13 to 50 microns.

From the microscope measurements on twelve wool samples diameter-frequency distribution curves are obtained for each sample. These data are used to illustrate the wide dispersion of diameters which are collected into a single averaged result by the eriometer. On the average, this range of variation extends to $\pm$ 40 per cent. from the mean with an average deviation of roughly 10 per cent. The diffraction method is not suitable for the averaging of all possible diameter distributions. The diameter distribution should not cover a wide range of variation combined with a frequency curve which is very flat or irregular. For best results there should be a pronounced concentration of diameters about a single class.

It is shown that the precision and accuracy of the eriometer are quite sufficient for the intended purpose. The eriometer averages for twelve wool samples all fall within 2 microns of the corresponding values obtained by averaging many individual determinations with the microscope.

An investigation is made of sources of error arising from irregularities in form and orientation of the fibers, such that the theoretical condition of parallelism between the fibers is

not fulfilled closely. This possibility of error may be reduced to negligible proportions by reasonable care in the mounting of the fibers and is not regarded as of serious practical importance.

Further developments in the construction of the eriometer are outlined by which a larger area of the sample may be observed and some quantitative information on the dispersion of diameters obtained.

ALPHA CELLULOSE CONTENT AND COPPER NUMBER OF PAPER.

The alpha cellulose content of cellulose materials is determined by subjecting the cellulose to the action of sodium hydroxide of mercerizing strength for a definite period of time. The copper number is the weight of copper, in grams, reduced from an alkaline copper solution by 100 grams of the cellulose material. Both of these determinations are of much value in determining the purity of cellulose materials and have been used by the bureau for determining the purity of the cellulose in paper in connection with its work on the permanence of paper.

However, paper is unlike most other cellulose materials in that the fibers are compressed into a dense, compact sheet and are protected by sizing materials. This formation makes reactions with chemical reagents slow and often incomplete. To avoid the possible errors caused by incomplete reactions the usual procedures for determining the alpha cellulose content and copper number of cellulosic materials have been modified so that they are better adapted for the analysis of paper.

The modification considered of prime importance is the reduction of the paper to a cotton-like form by mechanical disintegration. A grinder devised especially for this purpose will be described in detail in the April number of the *Bureau of Standards Journal of Research*. Accompanying data show the necessity for grinding papers before these tests are made.

The alpha cellulose method is based upon that of Jentgen with modifications suggested by Parsons and Ross and other modifications developed at the bureau. The paper includes data on the accuracy of the method and reproducibility of the

results. The percentage values are easily reproducible by different analysts within 0.3. Errors caused by the amounts of sizing materials in the paper are pointed out and the procedure to avoid them is outlined. A comparison of the results obtained with this modified procedure was made with the results obtained with the tentative method of the cellulose division of the American Chemical Society. The two methods agree very well when the test specimen is in the disintegrated form in both.

The copper number method is based upon the method of Braidy and Knecht and Thompson with a few modifications developed at the bureau. Most copper number methods employ Fehling solution as the copper containing reagent, but these methods are subject to inaccuracies caused by the autoreduction of the solution and by the alteration of the cellulose by the high alkalinity. The Braidy method replaces the sodium hydroxide of the Fehling solution with sodium carbonate and sodium bicarbonate which is much less alkaline. The results of a comparison of the Braidy method with a method employing Fehling solution, formerly used at the bureau, show that the Braidy method has an average error one-third as large as the Fehling solution method. Results obtained by the Braidy method can be duplicated within one per cent.

Studies in Superconductivity. W. J. DE HAAS AND J. VOOGD. (Comm. Phys. Lab. University of Leiden, No. 199.) The authors had discovered previously that the metal gallium possesses the property of superconductivity. It is the sixth substance known to have this peculiarity. The gallium used in their former investigation contained a trace of indium. "As we did not know the part played by this admixture of indium in the occurrence of the superconductivity, we directly planned a repetition of the research with spectroscopically pure gallium." 160 mg. of the spectroscopically pure metal were obtained and used, a little rod having been prepared whose resistance was measured. Low temperatures were produced by using helium which boils at 4.°2 K. The following table shows the relation of resistance to temperature:

Temp. in ° K.	Pressure in mm.	Resistance in Hg. Ohms.
4.21	770	0.000608
1.38	2.01	0.000599
1.09	0.274	0.000593
1.07	0.243	0.00

"The total fall of resistance takes place between 1°.10 K. and 1°.07 K."

The compound Bi_5Tl_3 is known to become superconducting above the boiling point of helium. The resistance of a rod of this substance was measured at five different temperatures when the intensity of the transverse magnetic field applied by a Weiss type electromagnet was varied. At any temperature from 4.0° K. to 3.°39 K. an increase in the magnetic intensity produced an increase in resistance. Thus at 4°.2 K. the following relations were found to exist:

Intensity in gauss	3865	4038	4151	4181	4254	4634
Resistance in ohms	0	0.000221	0.000436	0.000496	0.000582	0.000597

The magnetic intensity for which the value of the resistance is zero depends on temperature being greater as the temperature is lowered. Thus,

Intensity for zero resistance in gauss	3865	4149	4402
Temperature, ° K.	4.2	4.03	3.88

In general the behavior of the eutectics of the systems Sn-Bi and Sn-Cd is similar to that just described.

"We found that at 3°.4 K. a field of 5.3 kilogauss did not yet disturb the superconductivity of Bi_5Tl_3. We have not yet extended our investigations to still lower temperatures. An extrapolation however of the data would predict that at a temperature of 1°.3 K. a field of about 9 kilogauss would not yet disturb the superconductivity. *This would render possible the production of magnetic fields of this order of magnitude with a solenoid from Bi_5Tl_3 wire without production of heat.*" G. F. S.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING MARCH 18, 1931.

The regular monthly meeting of The Franklin Institute was called to order at eight-fifteen by the President of the Institute, Mr. Hayward, who presided.

He immediately called upon the Secretary, who stated that the minutes of the important business meeting of February had been printed in full in the March JOURNAL and moved that they be approved as printed. This motion was seconded and adopted unanimously.

The Secretary then announced that there was no other business to be presented at this meeting.

The President introduced the speaker of the evening, Dr. Walter R. Ingalls, who talked on the subject, "The Wealth of Nations, with Especial Reference to That of the American People." Dr. Ingall's lecture was a very challenging and stimulating one and aroused a considerable amount of highly interesting discussion.

The meeting adjourned at nine-thirty-five p.m., after an expression of keen appreciation by the President.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, March 11, 1931.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 11, 1931.

MR. HENRY B. ALLEN *in the Chair.*

The following reports were presented for final action:

No. 2856: Process for Treating Petroleum Hydrocarbons.

This report recommended the award of the John Price Wetherill Medal to Mr. Thomas Tarvin Gray, of Newark, New Jersey, "In consideration of his pioneer work in the application of a catalytic process to the refinement of gasoline."

No. 2929: Telephone Transmitter.

This report recommended the award of the John Price Wetherill Medal to Doctor Edward C. Wente, of the Bell Telephone Laboratories, Inc., New York City, "In consideration of his contributions to the design and construction of condenser transmitters."

No. 2935: Literature.

This report recommended the award of the Louis Edward Levy Medal to

Doctor J. Stuart Foster, of McGill University, Montreal, Canada, for his paper entitled "Some Leading Features of the Stark Effect," which appeared in the May, 1930 issue of the JOURNAL OF THE FRANKLIN INSTITUTE.

The following reports were presented for first reading:

No. 2919: Work of Dr. J. B. Whitehead in Dielectrics.
No. 2937: Clark Medal.
No. 2938: Henderson Medal.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, March 18, 1931.*)

RESIDENT.

DR. J. BENNETT HILL, Chemist, The Atlantic Refining Company, 3144 Passyunk Avenue, Philadelphia, Pennsylvania. For mailing: 402 Wynmere Road, Wynnewood, Pennsylvania.

DR. HOWARD B. HOLROYD, Research Engineer and Physicist, Westinghouse Electric & Manufacturing Co., Lester, Pennsylvania.

MR. PHILIP X. RICE, Electrical Engineer, Leeds and Northrup Company, For mailing: 704 East Willow Grove Avenue, Chestnut Hill, Pennsylvania.

NON-RESIDENT.

DR. POL N. CORYLLOS, Surgeon, A.B. M.S. M.D., 48 East Seventy-fifth Street, New York City. For mailing: 812 Park Avenue, New York City.

CHANGES OF ADDRESS.

MR. ALFRED E. BLAKE, 1511 Dickinson Road, West Brookline, Upper Darby Post Office, Pennsylvania.

MR. GEORGE RAYMOND HALL, Sixteenth Floor, 135 William Street, New York City, New York.

LINN HELANDER, ESQ., 4604 Bayard Street, Pittsburgh, Pennsylvania.

MR. JONATHAN JONES, c/o McClintic-Marshall Corporation, Bethlehem, Pennsylvania.

DR. GEORGE L. KELLEY, c/o Pressed Steel Company of Great Britain, Ltd., Cowley, Oxon, England.

OTTO V. KRUSE, ESQ., c/o I. P. Morris & De La Vergne, Inc., Richmond & Norris Streets, Philadelphia, Pennsylvania.

DR. WILLIAM ROGERS, JR., 953 Harrison Street, Philadelphia, Pennsylvania.

LORD ERNEST RUTHERFORD, Cambridge University, Cambridge, England

PROFESSOR GEORGE WINCHESTER, 6 Allee Mariguy, Le Perreux S. Marne, Seine, France.

MR. FRANK L. WRIGHT, Special Products Department, The Midvale Steel Company, Nicetown, Philadelphia, Pennsylvania.

NECROLOGY.

Hon. Sir Charles A. Parsons, Newcastle-on-Tyne, England.

LIBRARY NOTES.

RECENT ADDITIONS.

American Society for Testing Materials. Proceedings. Thirty-third Meeting. Volume 30, Parts 1–2. 1930.

Appleton's Dictionary of Machines, Mechanics, Engine-Work, and Engineering. New edition (containing many portrait engravings). Two volumes. 1867–68.

AUERBACH, F., AND W. HORT. Handbuch der physikalischen und technischen Mechanik. Band 4, 2 Hälfte. 1931.

BAKER, DONALD M., AND HAROLD CONKLING. Water Supply and Utilization. 1930.

Chimie & Industrie. Neuvième Congrès de Chimie Industrielle. 1930.

CHITTENDEN, RUSSELL H., The Development of Physiological Chemistry in the United States. 1930.

DANTZIG, TOBIAS. Number, the Language of Science. 1930.

FLAMMARION, CAMILLE. Études et Lectures sur l'Astronomie. Five volumes. 1867–1874.

GREGORY, J. W. The Elements of Economic Geology. No date.

GROSSMANN, MARCUS A., AND EDGAR C. BAIN. High Speed Steel. 1931.

GUILLEMIN, ERNST A. Communication Networks. Volume I: The Classical Theory of Lumped Constant Networks. 1931.

HAAS, ARTHUR. Quantum Chemistry. A Short Introduction in Four Non-Mathematical Lectures. Translated by L. W. Codd. 1930.

HAMLIN, SCOVILLE, Editor. The Menace of Overproduction: Its Cause, Extent and Cure. 1930.

HAYDEN, ARTHUR G. The Rigid Frame Bridge. 1931.

HECKEL, GEORGE B. The Paint Industry: Reminiscences and Comments. 1931.

HISCOX, GARDNER D., Editor. Henley's Twentieth Century Formulas, Recipes and Processes. New enlarged edition. 1930.

HOLLAND, HARVEY H. Avigation. First edition. 1931.

HOUSTOUN, R. A. A Treatise on Light. 1930.

HUBBARD, HENRY V., MILLER MCCLINTOCK, AND FRANK B. WILLIAMS. Airports: Their Location, Administration, and Legal Basis. 1930.

JONES, BRADLEY. Avigation. 1931.

KING, EDWARD SKINNER. A Manual of Celestial Photography. 1931.

KINSEY, V. EVERETT, AND THOMAS E. HOPKINS. Index to Iron and Steel Patents. Second edition. 1931.

KNOX, KATHARINE MCCOOK. The Sharples: Their Portraits of George Washington and His Contemporaries. 1930.

LAHEE, FREDERIC H. Field Geology. Third edition. 1931.

LAURSON, PHILIP GUSTAVE, AND WILLIAM JUNKIN COX. Properties and Mechanics of Materials. 1931.

LEIGHOU, ROBERT B. Chemistry of Engineering Materials. Third edition. 1931.

LEITH, C. K. World Minerals and World Politics. 1931.

LEVI-CIVITA, TULLIO. A Simplified Presentation of Einstein's Unified Fielp Equations. 1929.

MACKAYE, JAMES. The Dynamic Universe. 1931.

MAGUIRE, CHARLES JOSEPH. Aerology. A Ground School Manual in Aeronautical Meteorology. First edition. 1931.

MERRIMAN, MANSFIELD. Treatise on Hydraulics. Tenth edition, revised. 1916.

MERSEREAU, SAMUEL FOSTER. Materials of Industry: Their Distribution and Production. First edition. 1931.

MORRISON, L. H. American Diesel Engines. First edition. 1931.

MUNCH, JAMES C. Bioassays: a Handbook of Quantitative Pharmacology. 1931.

PAYNE, CECILIA H. The Stars of High Luminosity. 1930.

Philadelphia in 1824; or, a Brief Account of the Various Institutions and Public Objects in this Metropolis. 1824.

POLSON, J. A. Internal Combustion Engines. 1931.

PRESCOTT, SAMUEL CATE, AND CHARLES-EDWARD A. WINSLOW. Elements of Water Bacteriology with Special Reference to Sanitary Water Analysis. Fifth edition, revised. 1931.

RIES, H. Elementary Economic Geology. 1930.

Royal Society of London. Philosophical Transactions. Series A. Volume 229. 1930.

SCHUCHERT, CHARLES. Outlines of Historical Geology. 1931.

SEARS, WILLIAM P., JR. The Roots of Vocational Education. 1931.

Second World Power Conference, Berlin. Transactions. Volumes 1–19. 1930.

SHAPLEY, HARLOW. Flights from Chaos. 1930.

SMITH, ELVINO V., Publisher. Philadelphia, Camden and Suburbs. Map with street index and an analysis of all properties in Philadelphia. No date.

TAYLER, A. J. WALLIS. Industrial Refrigeration, Cold Storage, and Ice-Making. Being the seventh edition, revised and enlarged of "Refrigeration, Cold Storage, and Ice-Making." 1929.

TAYLOR, HUGH S. Elementary Physical Chemistry. 1928.

TAYLOR, HUGH S., Editor. A Treatise on Physical Chemistry: a Coöperative Effort by a Group of Physical Chemists. Second edition. Two volumes. 1931.

VOSKUIL, WALTER H. Minerals in Modern Industry. 1930.

WEICK, FRED E. Aircraft Propeller Design. First edition. 1930.

WOODMAN, A. G. Food Analysis. Typical Methods and the Interpretation of Results. Third edition. 1931.

WOODWORTH, JOSEPH V. Punches, Dies and Tools for Manufacturing in Presses. Fourth enlarged edition. 1930.

Zeitschrift für Instrumentenkunde. Generalregister für die Jahrgänge 31 bis 50 (1911–1930) und für die Deutsche Mechaniker-Zeitung Jahrgänge 1911 bis 1920. 1930.

BOOK REVIEWS.

ESSENTIALS OF ORGANIC AND BIOLOGICAL CHEMISTRY. By E. Wertheim, Ph.D., Professor of Chemistry, University of Arkansas. IV–175 pages, illustrations, 8vo. Easton, The Chemical Publishing Co., Inc., 1931.

Like many other textbooks of the type, this one had its inception in a course of lectures delivered by the author. Unlike many of the others, this book merely skims the purely technical side of the subject. Its text must have formed the basis of a very popular course in organic and biological chemistry. The introductions to the various elemental and complex organic compounds are quite informal and after being acquainted as to the rôle played by these compounds in everyday affairs, the reader feels like exclaiming: "So that is what you are; you do have a familiar appearance; I feel I have known you all my life."

The subject matter has been presented in its logical sequence beginning with methane, the simplest compound then climbing to such complex but well defined substances as sugars, local anesthetics, soporifics, antiseptics, dyes, hormones such as adrenalin and thyroxin. Continuing beyond these, we come to compounds which, up until now, have successfully withheld their ultimate composition from the pryings of ardent investigators. These are the complex carbohydrates, starches and celluloses; proteins of unimaginable variety and composition; and lastly, hormones, vitamins, enzymes, hemoglobin and chlorophyll, all of which play important catalytic rôles.

The book concludes with a very interesting but general description of the digestion processes, the measurement of metabolic rates, the calorific values of various foods and their particular appropriateness as necessary articles of diet. In regard to the inorganic elements, most of us suspected that there was phosphorus in the brain, calcium in the bones, iron in the blood, and iodine in the thyroid, but how many knew we had silica in our eye (source of the stony glare?), manganese in the pancreas, copper, nickel and cobalt in the blood, zinc in the liver and arsenic distributed apparently without rhyme or reason.

T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 368. A New Chart for Estimating the Absolute Ceiling of an Airplane, by Walter S. Diehl, 7 pages, illustrations, quarto. Washington, Government Printing Office, 1930, price ten cents.

This report, which was prepared for publication by the National Advisory Committee for Aeronautics, is concerned with the derivation of a chart for estimating the absolute ceiling of an airplane. This chart may be used in conjunction with the usual curves of power required and power available as an accurate substitute for extended calculation, or it may be used in the estimation of absolute ceiling when power curves are not available.

Bibliography of Aeronautics, 1929, compiled by Paul Brockett, 242 pages, Washington, Government Printing Office, 1930, price thirty-five cents.

This Bibliography of Aeronautics for 1929 covers the aeronautical literature

published from January 1 to December 31, 1929. The first Bibliography of Aeronautics was published by the Smithsonian Institution as Volume 55 of the Smithsonian Miscellaneous Collections and covered the material published prior to June 30, 1909. Supplementary volumes of the Bibliography of Aeronautics for the subsequent years have been published by the National Advisory Committee for Aeronautics. They are the Bibliography of Aeronautics for the years 1909–1916, 1917–1919, 1920–1921, 1922, 1923, 1924, 1925, 1926, 1927, and 1928.

As in previous volumes, citations of the publications of all nations are included in the languages in which these publications originally appeared. The arrangement is in dictionary form with author and subject entry, and one alphabetical arrangement. Detail in the matter of subject reference has been omitted on account of the cost of presentation but an attempt has been made to give sufficient cross reference for research in special lines.

FLIGHTS FROM CHAOS: A Survey of Material Systems from Atoms to Galaxies. By Harlow Shapley, Ph.D., LL.D., Director of the Harvard College Observatory and Paine Professor of Astronomy, Harvard University. vii–168 pages, 8vo. New York, Whittlesey House, 1930. Price $2.50.

To classify, in the brief compass of an ordinary sized book, all of creation from the light quanta and electrons on the one hand, up through man himself, to the entire space-time universe at the other end of the scale, is an ambitious undertaking. Yet that is just what Dr. Shapley has attempted in this work, originally presented as a series of lectures at the College of the City of New York, and the result is surprisingly good. Naturally, one might easily find things to object to, but after all any classification is largely a matter of personal opinion, to which anyone has a right. In any event, Dr. Shapley has produced, it seems to this reviewer, a reasonable scheme of arrangement of the systems of the universe. This scheme, as well as a wealth of information about the items it comprehends, is given in the author's usual fascinating and lucid style, simply enough written to appeal to the layman, yet with all the authority of the prominent position that he holds. It should prove especially helpful to the person, whether layman or scientist, who finds himself bewildered by the modern discoveries in astronomy and physics.

Dr. Shapley divides the material systems into 19 classes, ranging upwards and downwards from the group including man, which is numbered 0, and designated as "Colloidal and Crystallic Aggregates." The lowest class he numbers -4, and calls it "Corpuscles," including in it the sub-classes of "Light Quanta," "Electrons" and "Protons," as well as a blank space for possible smaller units that may be discovered. Also, "to register our conviction that the series of systems extends downwards, and that for us its limit is indeterminate," he places at the bottom an unnamed and blank class, numbered -5.

Going upwards, he suggests -3, "Atoms"; -2, "Molecules"; -1, "Molecular Systems" and then 0, "Colloidal Aggregates." The latter, like many of the others, is subclassified. One of the two subclasses is "Inorganic" and the other, including the entire living world, "Organic." Continuing, he has 1, "Meteoritic Associations"; 2, "Satellitic Systems"; 3, "Planetary Structures" (the stars); 4, "Double and Multiple Stars"; 5, "Galactic Clusters;"

6, "Globular Clusters"; 7, "Galaxies"; 8, "Multiple Galaxies"; 9, "Supergalaxies"; 10, "The Metagalaxy" (which he defines as "the vague supersystem—all-comprehensive but incomprehensible" and which includes all of the lower groups in the scheme); 11, "The Cosmoplasma"; 12, "The Universe, or Space-Time Complex" and finally, for number 13, a blank and unnamed class corresponding to that of − 5 at the other end.

Perhaps the most irregular part of the whole classification is that of "The Cosmoplasma," which is something of a chaos in itself but which is, in some ways, the most important class of all. The "metagalaxy" includes, says the author, the galaxies and the supergalaxies, but "it does not include the random stars or the lost planets that have become detached from galaxies and rove through intergalactic space." Nor does it include "those comets or meteors that have been ejected from our planetary system," the material that makes up the dark nebulæ, or the clouds of calcium and other vapors between the stars, which reveal their presence through the spectroscope.

But all of this stuff must be placed, if the classification is to be complete, so the "Cosmoplasma" is called into being to accommodate what is left over from the other classes. "It is the matrix," Dr. Shapley explains, "in which planets, stars and galaxies rotate; it constitutes the sidereal medium in which material systems undergo organized motion and development." Far from being unimportant, it contains the major portion of the material of the universe. It is not only the debris of countless ages of stellar evolution, but the light and energy in which form their mass has been radiated away. If, somewhere, somehow, sometime, this energy is being retransformed into matter, and we are to be saved from the "heat death," the cosmoplasma is doubtless responsible.

Mechanically, the book is well printed, in an attractive type face, and properly bound. The numerous small sketches, based on astronomical subjects, are very interesting and relieve the monotony of an unillustrated work. Of course, they do not reveal as much to the lay reader as would half-tone plates from actual photographs. One minor point that the reviewer would like particularly to commend is the use of capital initial letters for Earth, Moon and Sun. This practice seems to be becoming general among astronomical writers, though it is not yet authorized by the dictionaries. Certainly Earth has as much right to a capital as Jupiter, for instance, or Sun as much as Sirius.

Unfortunately, attention must be called to one unpardonable fault—the book lacks an index. Possibly it was felt that the entire work is a classification, and that an index was therefore superfluous. It contains such a wealth of information about the various topics, however, that the absence of an index will greatly limit its usefulness. It is to be hoped that this defect will be remedied in another edition, and a good book made still better.

JAMES STOKLEY.

APPLICATIONS OF INTERFEROMETRY, by W. Ewart Williams, M.Sc., Lecturer in Physics in the University of London, King's College, 104 pages, 16mo. New York, E. P. Dutton and Company, Inc.

This little book is a mine of information. It is valuable not only to students of optics but to chemists and engineers who are every day more and more making use of interference methods for solving problems in which a high degree of accuracy is required.

The book is divided into 6 chapters. The interference methods are divided into two classes, one in which there is a separation of wave-fronts, usually called diffraction, and the other in which there is a separation of amplitude, usually called interference. While the discussion of the various methods is brief it is accurate and up-to-date and it is logically developed.

The book should prove valuable to anyone interested in the technical uses of interferometers.

JAMES BARNES.

NUMERICAL MATHEMATICAL ANALYSIS. By James B. Scarborough, Ph.D., Associated Professor of Mathematics at the U. S. Naval Academy. XIV–416 pages, 8vo. cloth, Baltimore, The Johns Hopkins Press, 1930, Price $5.50.

If there are any gaps in the abundant supply of literature on applied mathematics which modern progress has made available for an endless variety of purposes, Dr. Scarborough seems to have found one. There are certainly very few texts in English, if any, that deal exclusively with the methods of analysis and numerical limitations of various mathematical processes which are employed in every activity of numerical evaluation. Without laying particular stress on computation forms, the work deals with these methods and their fundamental principles. The major topics include methods of interpolation, numerical differentiation and integration, numerical algebraic and transcendental equations, least squares, and harmonic analysis. These are merely general divisions, most of which are greatly subdivided.

The treatment is practical as well as logical. A general chapter is first given on the errors of a final result of the approximate figures with which a computer always deals, as distinguished from the exact figures of text-books of arithmetic. Thus the reader approaches the subject with no misunderstandings of the kind of results that may be expected from the processes which follow. The many detailed explanations which are given will be especially welcomed by the reader, particularly if he is studying the subject without the aid of a teacher. Another feature which will be appreciated by those readers who are not trained mathematicians is the absence of the use of notations peculiar to certain subjects and in the derivation of interpolation formulas, the application of operator symbols. With the various interpolation formulas, unusually full explanations of where and why they are to be used are given with fully worked out numerical examples. These examples, it may be mentioned, provide quite adequately illustrations of proper form for recording the steps in making computations.

There is no preponderance of thoroughness and lucidity of any one part of the work over another. Those who are interested in the construction of tables will find a strong appeal in the deductive and exhaustive character of the treatment of interpolation. Many in the ranks of machine and structural designers to whom Simpson's Rule is a household term will find the account of quadrature most illuminating, and broadening to his view of the subject of approximate integration.

In the division relating to the solution of numerical algebraic and transcendental equations we find the analysis and application of several well-known methods of evaluating roots, the decomposition of the equation into separate equations of a straight line and a curve whose simultaneous values determined graphically give a

root, the regula falsi, the solution by repeated plotting to a larger scale, the Newton-Raphson method of analytically approaching the root by means of repeated tangents which approach the curve at the root, the iteration process of repeated substitutions, and Graeffe's root-squaring method. The last, which is considered very fully, depends upon transforming the equation into one whose roots are high powers of those of the original equation. Simultaneous equations are dealt with by the Newton-Raphson method and the iteration process. Horner's method is omitted as a familiar subject of algebra. The Horner method is a rather ponderous affair when the root is required to a considerable number of significant figures and requires a certain degree of chirographic workmanship to avoid the danger of misplaced figures. Those who struggled with it when they studied algebra will be grateful for more comfortable processes. A very valuable feature which appears throughout the work will be found in the thorough consideration which is given of the possible error of a result; that is, the maximum possible difference between the true value and the result of the computation.

The account of the methods for the solution of differential equations and the meticulous analysis which they receive constitute a veritable treatise on the solution of these equations with numerical coefficients and given initial conditions. In several chapters which may be generalized as probability and least squares, the author has developed an analysis of that important subject with its reduction to readily understandable numerical values with profuse explanations of all matters that need clarifying, that should inspire the reader to a sustained effort in pursuing the subject. Here as in other parts of the work, the text appears to anticipate every difficulty the reader may meet in following the argument. Harmonic analysis with numerical examples worked out at length have a final chapter. A table of values of the probability integral from 0 to 1.00000 is appended.

The work should prove an indispensable compendium to the computer and a most valuable text to the student who seeks sound methods in numerical mathematical processes.

L. E. P.

THE DEVELOPMENT OF PHYSIOLOGICAL CHEMISTRY IN THE UNITED STATES. By Russell H. Chittenden, Professor of Physiological Chemistry in Sheffield Scientific School of Yale University, 1882–1922. American Chemical Society Monograph Series. 427 pages, 8vo. The Chemical Catalog Company, Inc., 1930. Price $6.00.

This immensely interesting book is one of the more recent members of that steadily growing family of American Chemical Society Monographs. A chronological exposition on the development of physiological chemistry in the United States, it makes a somewhat attractive variation from the general run devoted to the greater details of narrower fields in chemistry.

It is indeed appropriate that Russell H. Chittenden should be the author of such a book. As one of the pioneers in his field, he had lived and worked in one of the great centers of physiological chemistry in the country. As a teacher and research worker he has kept a finger on the strengthening pulse of this rapidly growing offspring of chemistry and medicine. Beginning with the return of Henry P. Bowditch to America in 1871 and his establishment of the "first physio-

logical laboratory for the use of students in United States" at Harvard Medical School the author has closely followed the development of knowledge regarding the chemical aspects of things physiological as well as the extension of its centers for research and instruction throughout all the land.

One cannot read about these classic pieces of research without appreciating the sterling qualities of the workers and without experiencing a quickened interest in what is being told and looking ahead for that still to come.

In passing, one recalls the stupendous work of Reichert and Brown on the crystallography of hemoglobins. This involved the making of 600 microphotographs of the blood of a great variety of mammals. Other outstanding bits of research sticking in the mind are: the work on the physical chemistry of the blood, the chemical composition of brain matter, the chemical study of the Tubercle Bacillus, the discovery of vitamins and their specific roles in maintaining the well being of the body and, most recently, the study of the hormones, secretions of the ductless glands, which now are shouldering the blame for making us what we are today.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

Lecons de Chimie Analytique, par Alcide Jouniaux, 350 pages, illustrations, 8vo. Paris, Librairie Scientifique Hermann et Cie, 1931, Price 60 frs.

Tables Annuelles de Constantes et Données Numériques, Donnees Numeriques sur les Colloides et L'Adsorption, Redigées par G. Génin, Extrait du Volume VII, 1925–1926, pages 1398 to 1454, quarto, Paris, Gauthier-Villars et Cie, 1930. Données Numériques d'Électricité Magnétisme et Électrochimie, Redigées par A. Buffat, L. Durand, G. Foex, R. L. Fortrat, N. Marinesco, et N. Thon, Extrait du Volume VII, 1925–1926, pages 928 to 1397, quarto, Paris, Gauthier-Villars et Cie, 1930. Données Numériques de Radioélectricité, Redigées par R. Mesny, Extrait du Volume VII, 1925–1926, pages 1020 to 1037, quarto, Paris, Gauthier-Villars et Cie, 1930.

Le Génie Civil, Numéro Spécial Publié a L'Occasion du Cinquantenaire de la Foundation du "Génie Civil," L'évolution et le développement des principales industries depuis cinquante ans, 136 pages, illustrations, quarto. Paris, Administration & Redaction.

Le Moniteur des Travaux Publics de L'Entreprise & de L'Industrie, Numéro Spécial L'Esthétique des Ponts Bois, Pierre, Métal, Béton & Béton Arme, par Jacques Pilpoul, préface de M. Édouard Arnaud, appendice de M. Camille Bouché, 94 pages, illustrations, quarto. Paris, Le Moniteur des Travaux Publics, 1931, Price 30 frs.

National Advisory Committee for Aeronautics. Bibliography of Aeronautics 1929, 242 pages, 8vo. Washington, Committee, 1930. Price 35 cents. Technical Notes, No. 365, Interference Effects and Drag of Struts on a Monoplane Wing, by Kenneth E. Ward, Langley Memorial Aeronautical Laboratory, 12 pages, tables, plates, quarto, Washington, Committee, 1931. No. 366, Torsion in Box Wings, by John B. Wheatley, 44 pages, illustrations, quarto, Washington, Committee, 1931. No. 367, The Aerodynamic Characteristics of Three Tapered

Airfoils Tested in the Variable Density Wind Tunnel, by Raymond F. Anderson, Langley Memorial Aeronautical Laboratory, 12 pages, tables, illustrations, quarto, Washington, Committee, 1931.

U. S. Coast and Geodetic Survey. Results of Observations made at the United States Coast and Geodetic Survey Magnetic Observatory at Vieques, P. R. in 1923 and 1924, by W. N. McFarland, 94 pages, tables, diagrams, Washington, Government Printing Office, 1931. Price 40 cents.

National Research Council of Canada. The Maintenance of a Standard of Electromotive Force: Notes on Standard Weston Cells, by A. Norman Shaw and H. E. Reilley, pages 473 to 489, 8vo. Reprinted from the Canadian Journal of Research, 1930.

Royal Society of London, Philosophical Transactions. The Ageing of Standard Cells: Increased Accuracy in Their Use: and International Comparisons, by A. Norman Shaw, H. E. Reilley and R. J. Clark, pages 125 to 162, tables, quarto. London, Harrison & Sons, Ltd. n.d.

CURRENT TOPICS.

An Optical Method of Measuring the Mercury Content of the Air. K. MÜLLER AND P. PRINGSHEIM (*Naturwissenschaften*, April 25, 1930.) Stock in recent years has shown that cases of chronic mercury poisoning are much more common than had been believed. In the long run exceedingly small quantities of mercury vapor exert an injurious effect on the health. At ordinary temperatures the pressure of saturated mercury vapor amounts to only about .001 mm. of mercury; this corresponds to about 10 mg. of the liquid to 1 m.^{3} of air. Stock holds that even 1/1000 of this concentration is to be shunned in rooms where people are for long stretches of time. To determine the quantity of mercury in the air of a room he pumped .3 m.^{3} of the air through a U-tube jacketed with liquid air. Liquid mercury condensed on the walls and its quantity was subsequently obtained in the laboratory. The process was lengthy, because five hours were required for the passage of the air and, besides, it required a pump, a gas-meter and large amounts of liquid air. By it, however, it is possible to detect as small a concentration as .0001 mg. per m.^{3} The General Electric Co. has developed an indicator for mercury vapor depending on the blackening of selenium sulphide. It is very convenient to use but can scarcely be employed for concentrations of less than .5 mg./m.^{3}

The principle of the method devised by the authors is this. Mercury vapor absorbs a certain line emitted by the mercury arc, the resonance line of wave-length 2537 A. In the ultra-violet the strength of the absorption depends on the breadth of the line, the length of the absorbing air path, the pressure of the mercury vapor in the air and also to some extent on the pressure of other gases present. Therefore for purposes of measurement there must be a source of light that emits the line with a fixed width and the degree of absorption of this line must be found by experiment for quantities of mercury vapor per m.^{3} varying from 10 to .005 mg. In the actual apparatus evolved the light from a water-cooled mercury arc lamp was focused on a resonance lamp which emits resonance light. The 2537 A line comes from the second lamp with its width lessened and it is precisely the narrow spectral region of its width that is absorbed by mercury vapor in the air. A part of the light from the resonance lamp traverses a tube containing the sample of air to be investigated and falls on the cathode of a photoelectric cell. This cathode is joined to an electrometer whose deflection is

directly proportional to the intensity of the light incident on it. By comparing the intensity of the light after passing through a tube containing mercury vapor with that when no vapor is present it is possible to determine the amount of vapor in the tube. To avoid variations in the intensity of the ultra-violet light used a zero method was applied in which a second portion of the beam from the resonance lamp went to a second photoelectric cell joined to the same electrometer in such a way as to produce zero deflection when the two paths traversed by the radiation were alike in absorption. By substituting air containing mercury vapor for pure air in one of the paths a deflection was caused and from it the absorption was inferred. Since benzol and its derivatives show marked absorption in the region of the 2537 A line a method is provided for distinguishing absorption by organic compounds from mercury absorption.

To test the air in a room an evacuated tube is brought into it and a cock is opened for the influx of a sample of the air and then closed. The tube is taken to the apparatus and its absorption is measured. For the analysis by absorption only two or three minutes are required. The drawbacks are the complexity and high cost of the apparatus and the necessity of having trained physicists to operate it. The authors applied their apparatus to a study of the mercury content of the air in a Berlin laboratory. The content proved to be very different in different rooms. In some that had been intentionally kept free from mercury contamination it was too small to be measured. "In most laboratories in which work had been regularly done with mercury pumps and manometers its value lay between .02 and .06 mg./m.^{3} In one room where to all appearances generations of students preparing for the doctor degree had spilled mercury it exceeded the value .5mg./m.^{3} In all spaces the mercury content showed exceeding great variations with time." After airing a room it was practically zero. Even in the room just mentioned it dropped to about 1/17 of the value above given. Repeated opening and shutting of doors into halls brought the content down whereas, on the contrary, it was made larger by the production of air currents by local heating or otherwise. In one room the functioning of a mercury diffusion pump for an hour elevated the content from .008 to .015 mg. per m.^{3} G. F. S.

Changes in the Vapor Pressure of Liquids due to the Curvature of their Surfaces. MANFRED THOMÄ. (*Zeit. für Physik*, Vol. 64, page 224.) Sixty years ago Kelvin drew the conclusion that the vapor pressure over a concave surface of a liquid is less than over a flat surface and that the pressure at a flat surface is less than over a convex surface. His argument was based on very simple considerations, and led to the result

$$dp = T + \frac{d'}{d' - d} \cdot \frac{2}{r},$$

where dp is change of pressure, T is surface tension, d is density of liquid and d' is density of the saturated vapor r is the radius of the tube in which liquid stands. It seems that no experimental verification of this formula has hitherto been undertaken. Qualitative confirmation is, however, furnished by the fact that in a mixture of large and small drops the small ones evaporate while the large ones grow in size.

To test the formula, the space on one side of a very thin membrane was joined to a vessel in which was a flat surface of liquid while the other side was in communication with another vessel containing a concave surface of the same liquid. The membrane was made by dropping a solution of collodium on mercury and letting the solvent evaporate. When water was used small quantities of air were attached to the film and subsequently broke it. The thickness of the membranes used was about 1.5×10^{-4} cm. The slight bulging of the membrane due to excess of pressure on one side was measured by counting the interference fringes formed by a fixed mirror and another attached to the membrane.

As the experimental liquid valeric acid was selected, $(CH_3)_2$-$CHCH_2COOH$, because it possesses a suitable vapor pressure and, further, scarcely acts on the substance used to mount the membrane. The number of fringes seen to be displaced never exceeded 18.1.

In the formula above given the surface tension, and both densities are unchanged during a set of experiments. Hence the product of radius by change of pressure equals a constant. In a set of experiments at 14.4° C. these products ranged from .400 to .356, and in another set at 13.6° from .394 to .368.

The change of pressure indicated by the passing of one fringe was calculated from experimental data to be equal to 2×10^{-4} dynes per sq. cm. or 1.5×10^{-7} mm. of mercury or, again, 2×10^{-10} atmosphere. It is found that this result of calibration gives values for the change of pressure that are in fair agreement with those calculated by means of Kelvin's formula. The maximum difference of pressure observed between a flat and a curved surface was about one-four hundred thousandth of a mm. of mercury. This was for the liquid in a tube about .2 mm. in radius.

G. F. S.

The Gyromagnetic Effect for Paramagnetic Substances. W. SUCKSMITH. (*Proc. Roy. Soc.*, A 807.) The view that the atom is

composed of electrons with negative electrical charges revolving about the central nucleus brought with itself the implication that the magnetic properties of the atom were due to these rotating charges. Since these charges may be regarded as resembling spinning tops with axes free to change direction it seemed legitimate to apply to an aggregate of atoms the laws known to hold for an aggregate of rotating tops. If a group of such tops be rotated as a whole, the individual tops tend to set their axes parallel to the general axis of rotation. By similar reasoning the axes of rotation of the electrons in a group of atoms, rotated as a whole, should strive to become parallel to the main axis of rotation. This manifests itself as an increase in the magnetic moment of the rotated specimen. This was tried with magnetic substances and the result qualitatively verified the prediction. The converse of this is that the alignment of the axes of rotation of the electrons by the application of a magnetizing force should produce a change in angular momentum of the magnetized body, that is, should cause it to rotate. This too was tried and experiment again qualitatively verified prediction in the case of ferromagnetic bodies. "The angular momentum thus associated with the change of magnetic moment is very small, and has hitherto been measured only for ferromagnetic bodies. Experiments by various workers using different methods have shown that the angular momentum is within the limits of experimental error exactly one-half that expected on the orbital electron theory. This has been known as the gyromagnetic anomaly. Recently theoretical developments, however, indicate that the source of magnetic moment in ferromagnetic bodies is due to the spin of the electron." For the spinning electron the theory is in accord with the results of experiment for ferromagnetic substances. For paramagnetic substances there is reason to believe that both the spin and the motion of electrons in orbits make contribution to the magnetic moment. Moreover the relative proportions of the two contributions ought to be indicated by the size of the gyromagnetic ratio determined by experiment. The author therefore addresses himself to the difficult task of comparing angular momentum and magnetic moment in the case of a paramagnetic substance where under the best of circumstances only a feeble magnetism can be evoked.

He selects the method of producing rotation of the specimen by magnetizing it and, since the effect of the application of a single magnetizing impulse can be but small, he uses the resonance method of de Haas and Einstein in which the force is applied again and again, its frequency being the same as that of the oscillating system of which the paramagnetic material forms the chief part. The

dimensions of the oscillating system were designed to make possible the greatest amplitude caused by the recurrent application of the magnetizing force.

As the paramagnetic substance dysprosium oxide, Dy_2O_3, was selected. It is strongly paramagnetic and follows Curie's law. Its mass susceptibility amounts to 238×10^{-6} at 15° C. This substance was suspended in a vertical glass tube 6.8 cm. long that hung in the axis of the magnetizing coil. In this the direct magnetizing current was reversed by an ingenious device that permitted adjustment to .002 sec. To get rid of the damping effect of the air and thus to increase the maximum amplitude attainable by the resonance method the oscillating system was kept in air at a pressure of about 10^{-4} mm. mercury. Errors due to electrostatic and electromagnetic effects, to ferromagnetic impurities and to temperature changes were eliminated. It is really a noteworthy achievement to obtain rotation of a paramagnetic substance by the application of a magnetizing force.

In the theory of this effect a factor appears which is equal to 2 for spinning electrons and to 1 for electrons moving in orbits. From the author's experiments its value for the Dy^{+++} ion comes out as 1.28. This indicates that the magnetic moment is due to both spin and orbital constituents. The values of this factor are in good agreement with one another though derived from different experiments. G. F. S.

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